CHAPTER 2 TIDES

'. . . being governed by the watery Moon . . .'
Richard III, Act II, Sc. II.

The longest oceanic waves are those associated with the tides, and are characterized by the rhythmic rise and fall of sea-level over a period of several hours. The rising tide is usually referred to as the **flow** (or flood), whereas the falling tide is called the **ebb**. This ebb and flow of the tide is manifested in tidal currents (see Section 2.4.1), which can be very powerful, especially where the water is constrained by shallow depth or adjacent land masses. From the earliest times it has been realized that there is some connection between the tides and the Sun and Moon. Tides are highest when the Moon is full or new, and the times of high tide at any given location can be approximately (but not exactly) related to the position of the Moon in the sky. Because the relative motions of the Earth, Sun and Moon are complicated, it follows that their influence on tidal events results in an equally complex pattern. Nevertheless, the magnitudes of the tide-generating forces can be precisely fomulated, although the response of the oceans to these forces is modified by the permanent effects of topography and the transient effect of weather patterns.

2.1 TIDE-PRODUCING FORCES—THE EARTH–MOON SYSTEM

The Earth and the Moon form a single system, mutually revolving around a common centre of mass, with a period of 27.3 days. The orbits are in fact slightly elliptical, but to simplify matters we will treat them as circular for the time being. The Earth revolves eccentrically about the common centre of mass, which means that all points within and upon the Earth follow circular paths, all of which have the same radius (two examples are shown on Figure 2.1 as points C and X). Each point will also have the same angular velocity of $2\pi/27.3$ days. Because the angular velocities, and the radii of the circular paths travelled, are the same for all points, it follows that all points on and within the Earth experience an equal acceleration (the product of the radius and the square of the angular velocity) and hence an equal centrifugal force as a result of this eccentric motion.

centrifugal forces
solid body rotation

The eccentric motion has nothing whatsoever to do with the Earth's rotation (spin) upon its own axis, and should not be confused with it, nor should the centrifugal force due to the eccentric motion be confused with the centrifugal forces due to the Earth's spin which increase with distance from the axis, whereas those due to the eccentric motion are equal at all points on Earth.

why?

If you find this concept difficult, the following simple analogy may help. Imagine you are whirling a small bunch of keys on a short length (say 25 cm) of chain. The keys represent the Moon, and your hand represents the Earth. You are rotating your hand eccentrically (but unlike the Earth it is not spinning as well), and all points on and within your hand are

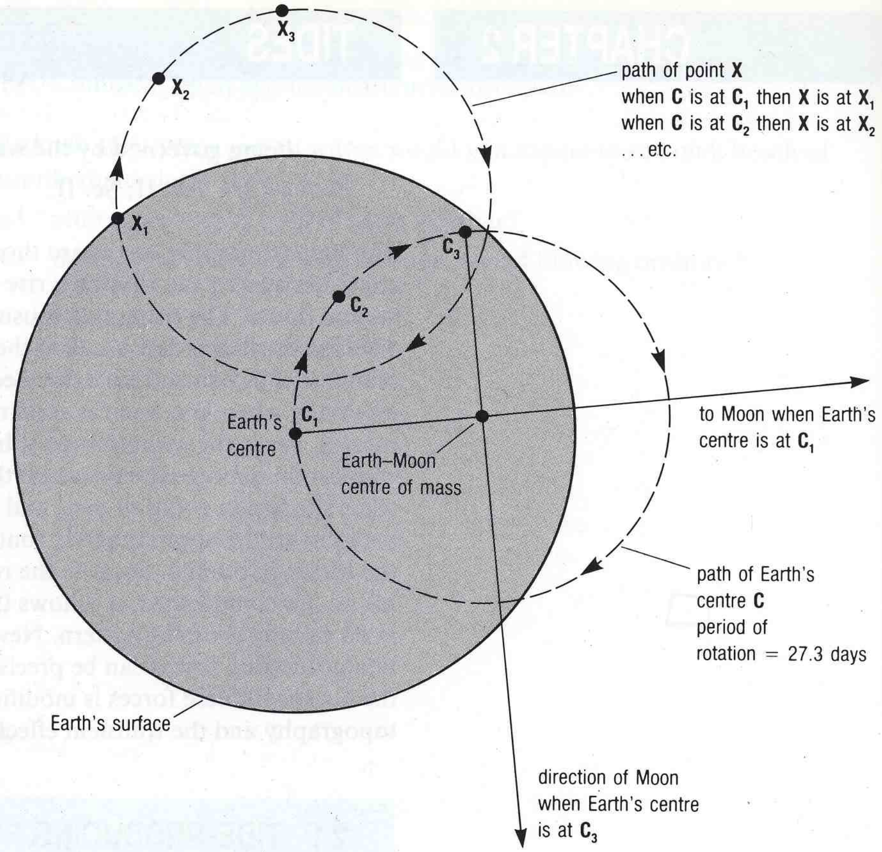

Figure 2.1 The eccentric revolution of the Earth about the Earth–Moon centre of mass, as viewed from above one of the poles, when the Moon is directly above the Equator. Every point on Earth follows a circular path analogous to those traced out by points C and X.

experiencing the same angular velocity and the same centrifugal force. Provided your bunch of keys is not too large, the centre of mass of the 'hand-and-key' system lies within your hand.

The total centrifugal force within the Earth–Moon system exactly balances the forces of gravitational attraction between the two bodies, so that the system as a whole is in equilibrium, i.e. we should neither lose the Moon, nor collide with it, in the near future. The centrifugal forces are directed parallel to a line joining the centres of the Earth and the Moon (see Figure 2.2). Now consider the magnitude of the gravitational force exerted by the Moon on the Earth. This will not be the same at all points on the Earth's surface, because not all these points are the same distance away from the Moon. So, points on the Earth nearest the Moon will experience a greater gravitational pull from the Moon than will points on the opposite side of the Earth. Moreover, the direction of the Moon's gravitational pull at all points will be directed towards the centre of the Moon, and hence, except on the line joining the centres of the Earth and Moon, will not be exactly parallel to the direction of the centrifugal forces. The resultant (i.e. the composite effect) of the two forces is known as the **tide-producing force**, and, depending upon its position on the Earth's surface with respect to the Moon, may be directed into, parallel to, or away from, the Earth's surface. The relative strengths and directions (not strictly to scale) of the forces involved are shown on Figure 2.2.

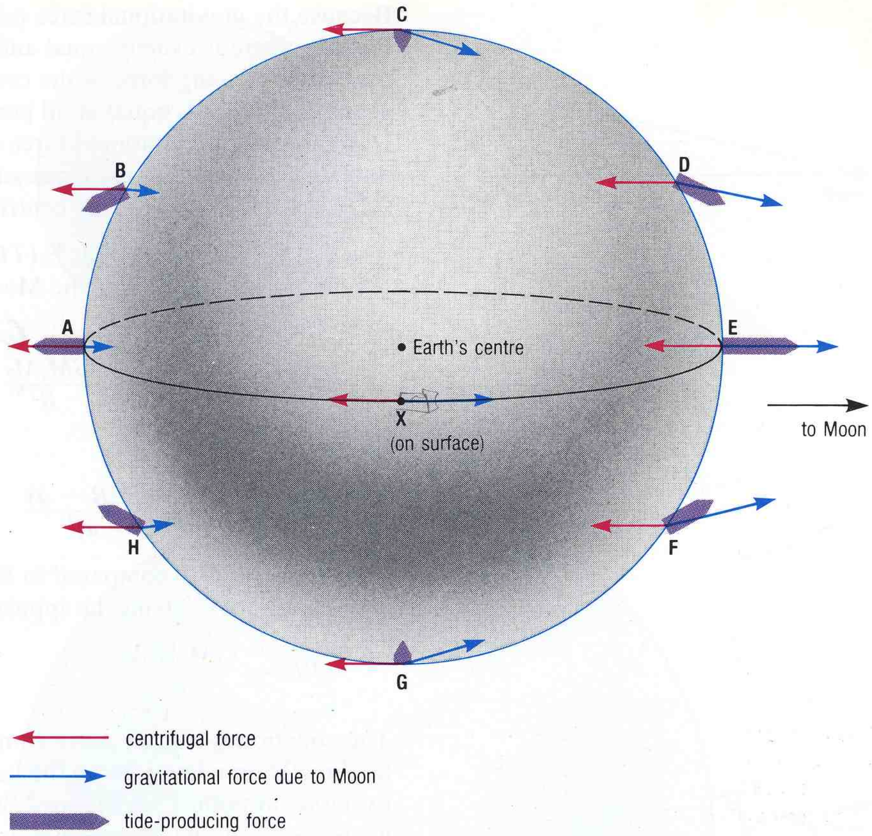

Figure 2.2 The derivation of the tide-producing forces (not to scale). The centrifugal force has exactly the same magnitude and direction at all points, whereas the gravitational force exerted by the Moon on the Earth varies in both magnitude (inversely with the square of the distance from the Moon) and direction (directed towards the Moon's centre, with the angles exaggerated for clarity). The tide-producing force at any point is the *resultant* of the gravitational and centrifugal forces at that point, and varies inversely with the cube of the distance from the Moon (see text).

QUESTION 2.1 What would be the direction and approximate magnitude (within the context of Figure 2.2) of the tide-producing forces at:

(a) a point on the Earth's surface represented by point X on Figure 2.2?

(b) the Earth's centre?

The gravitational force (F_g) between two bodies is given by:

$$F_g = \frac{GM_1M_2}{R^2} \tag{2.1}$$

where M_1 and M_2 are the masses of the two bodies, R is the distance between their centres, and G is the universal gravitational constant ($6.672 \times 10^{11}\,\mathrm{N\,m^2\,kg^{-2}}$).

You may be puzzled at how to reconcile equation 2.1 with the statement on Figure 2.2 that the magnitude of the tide-producing force exerted by the Moon on the Earth varies inversely with the cube of the distance. Consider the point marked E on Figure 2.3. The gravitational attraction of the Moon at E (F_{gE}) is greater there than that at the Earth's centre, because E is nearer to the Moon by the distance of the Earth's radius (a).

Because the gravitational force exerted by the Moon on a point at the Earth's centre is exactly equal and opposite to the centrifugal force there, the tide-producing force at the centre of the Earth is zero. Now as the centrifugal force is equal at all points on Earth, and at the Earth's centre is equal to the gravitational force exerted there by the Moon, it follows that we can substitute the expression on the right-hand side of equation 2.1 (i.e. GM_1M_2/R^2) for the centrifugal force.

The tide-producing force at E (TPF_E) is given by the force due to gravitational attraction of the Moon at E (F_{gE}) minus the centrifugal force at E, i.e.

centrifugal

$$TPF_E = \frac{GM_1M_2}{(R-a)^2} - \frac{GM_1M_2}{R^2}$$

which simplifies to:

$$TPF_E = \frac{GM_1M_2a(2R-a)}{R^2(R-a)^2}$$

Now a is very small compared to R, so $2R-a$ can be approximated to $2R$, and $(R-a)^2$ to R^2, giving the approximation:

$$TPF_E \approx \frac{GM_1M_2 2a}{R^3} \qquad (2.2)$$

The equation is slightly more complex for points on the Earth that do not lie directly on a line joining the centres of the Earth and Moon. For example, at point P on Figure 2.3(a), the gravitational attraction (F_{gP}) would be, to a first approximation:

$$F_{gP} = \frac{GM_1M_2}{(R - a\cos\psi)^2} \qquad (2.3)$$

The length $a\cos\psi$ is marked on Figure 2.3(a). (ψ is the Greek 'psi'.)

Before reading on, have another look at Figure 2.2, and consider at which of the lettered points on that Figure the local tide-producing force would have most effect on the tides.

You may have considered point E as your answer. Certainly, E is nearest to the Moon, and hence is one of the two points where the difference between the centrifugal force and the gravitational force exerted by the Moon is greatest. However, all the resultant tide-producing force is acting vertically against the pull of the Earth's own gravity, which happens to be about 9×10^6 greater than the tide-producing force. Hence the local effect of the tide-producing forces at point E is negligible. Similar arguments apply at point A, except that F_{gA} is *less* than the centrifugal force, and consequently the tide-producing force at A is equal in magnitude to that at E, but directed away from the Moon (see also Figure 2.3(b)).

At the lettered points B, D, F and H on Figure 2.2 (which lie on the small circles defined on Figure 2.3(a)), the effects of the tide-producing forces are greatest, because at each there is a large horizontal component (known as the **tractive force**) of the tide-producing force. It is the tractive forces which cause the water to move, because, although small compared with the Earth's gravitational field, this horizontal component is unopposed by any other lateral force (apart from friction at the sea-bed, which is negligible in this context). Figure 2.3(b) shows where on the

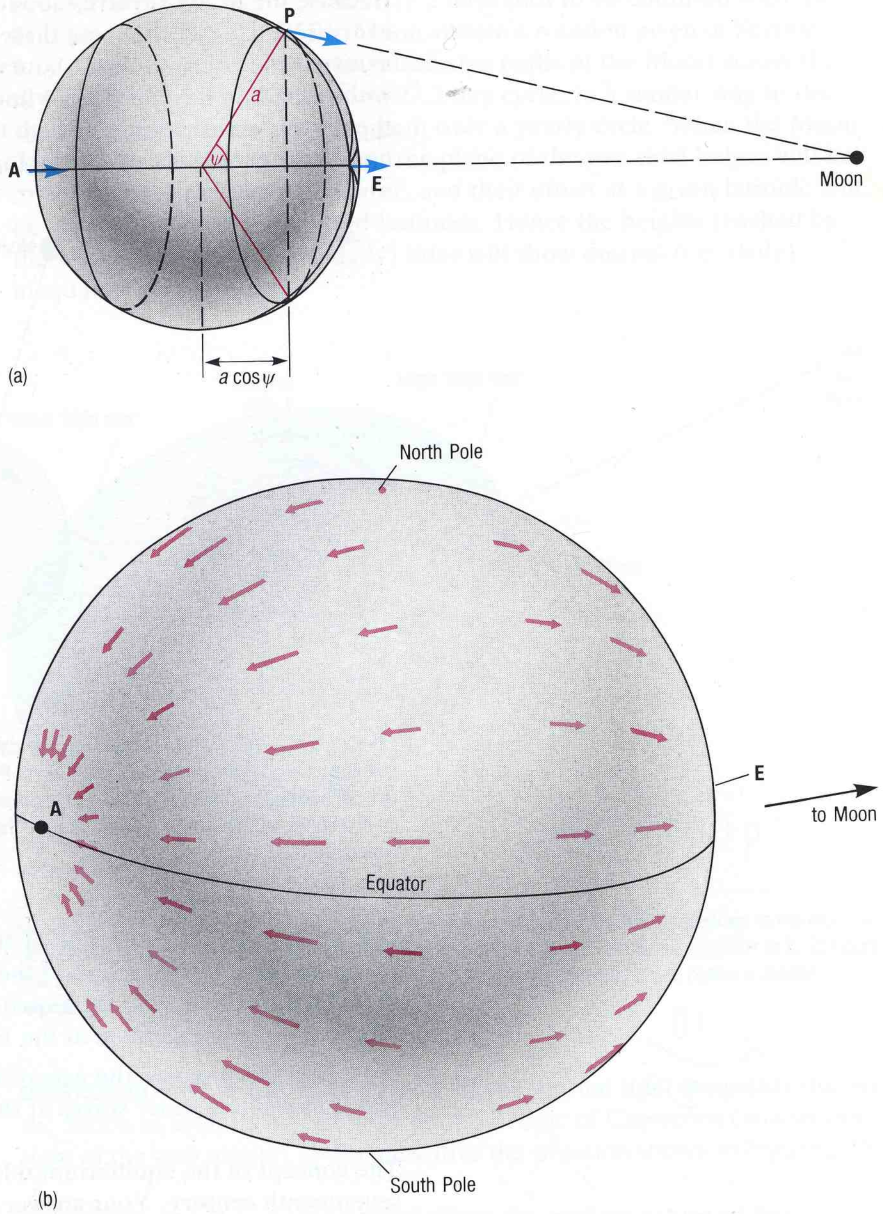

Figure 2.3(a) The effect of the gravitational force of the Moon at three positions on the Earth. The gravitational force is greatest at E (nearest the Moon) and least at A (furthest from the Moon). At P the gravitational force is less than at E, and can be calculated from equation 2.2. The tide-producing forces are smallest at A and E, but greatest at P, and all other points on the two small circles. The value for the angle ψ for these circles is 54°41′. The circles have nothing to do with latitude and longitude. For full explanation, see text.

(b) The relative magnitudes of the tractive forces at various points on the Earth's surface. The assumption is made that the Moon is directly over the Equator, i.e. at zero declination. Points A and E correspond to those on Figure 2.2.

Earth the tractive forces are at a maximum when the Moon is over the Equator. In this simplified case, the tractive forces would result in total movement of water towards points A and E on Figure 2.3(b). In other words, an equilibrium state would be reached (called the **equilibrium tide**), producing an ellipsoid with its two bulges directed towards and away from the Moon. So, paradoxically, although the tide-producing forces are minimal at A and E, those are the points towards which the water would go.

In practice, this ellipsoid does not develop, because the Earth rotates upon its own axis. The two bulges, in order to maintain the same position relative to the Moon, would have to travel around the world at the same rate (but in the opposite direction) as the Earth rotates with respect to the Moon.

Because the Moon revolves about the Earth–Moon centre of mass once every 27.3 days, in the same direction as the Earth rotates upon its own axis (which is once every 24 hours), the period of the Earth's rotation with respect to the Moon is 24 hours and 50 minutes (a **lunar day**). This is the reason why the times of high tides at many locations are almost an hour later each successive day (Figure 2.4).

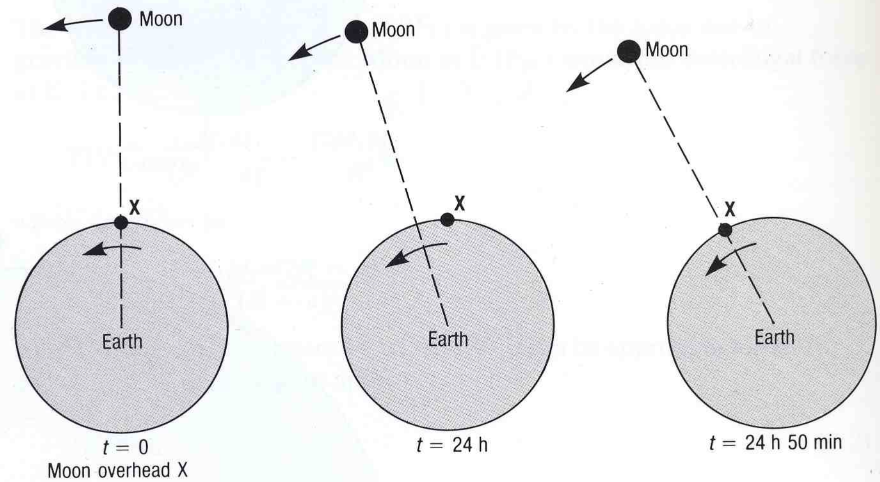

Figure 2.4 The relationship between a solar day of 24 hours and a lunar day of 24 hours and 50 minutes as seen from above the Earth's North Pole. Point X on the Earth's surface when the Moon is directly overhead comes back to its starting position 24 hours later. Meanwhile, the Moon has moved on in its orbit, so that point X has to rotate further (another 50 minutes' worth) before it is once more directly beneath the Moon.

$\frac{40000 \times 10^3 \text{ m}}{24 \times 3600}$

$\approx \frac{10^4}{24} = 9 \times 10^2 \frac{m}{s}$

$= \sqrt{gh}$

$16 \times 10^4 \frac{m}{s^2} = gh$

$16 \times 10^3 = h$

16000 m

QUESTION 2.2(a) Using a value of 40000km for the Earth's circumference, calculate the required speed of the tidal bulges in order to maintain an equilibrium tide around the Equator. (Assume for simplicity that the Moon is directly overhead at the Equator.)

(b) How deep would the oceans have to be to allow the tidal bulges to travel as shallow water waves at the speed you calculated in part (a)?

The concept of the equilibrium tide was developed by Newton in the seventeenth century. Your answer to Question 2.2 shows that, in practice, an equilibrium tide cannot occur at low latitudes on Earth, and, as will be seen in Section 2.3, the actual tides behave differently. However, the theoretical equilibrium tide demonstrates the fundamental periodicity of the tides on a semi-diurnal basis of 12 hours and 25 minutes.

2.1.1 VARIATIONS IN THE LUNAR-INDUCED TIDES

The relative positions and orientations of the Earth and Moon are not constant, but vary according to a number of interacting cycles. As far as a simple understanding of the tide-generating mechanism is concerned, only two cycles have a significant effect on the tides.

1 The Moon's declination

The Moon's orbit is not in the plane of the Earth's Equator, but is inclined at an angle of 28°. This means that a line joining the centre of the Earth to that of the Moon ranges up to about 28° either side of the equatorial plane, over a cycle of 27.2 days (not to be confused with the 27.3-day period of the Earth–Moon system's rotation given in Section 2.1). To an observer on Earth, successive paths of the Moon across the sky appear to rise and fall over a 27.2 day cycle, in a similar way to the variation of the Sun's apparent path over a yearly cycle. When the Moon is at a large angle of declination, the plane of the two tidal bulges will be offset with respect to the Equator, and their effect at a given latitude will be unequal, particularly at mid-latitudes. Hence the heights reached by the semi-diurnal (i.e. twice daily) tides will show diurnal (i.e. daily) inequalities (Figure 2.5).

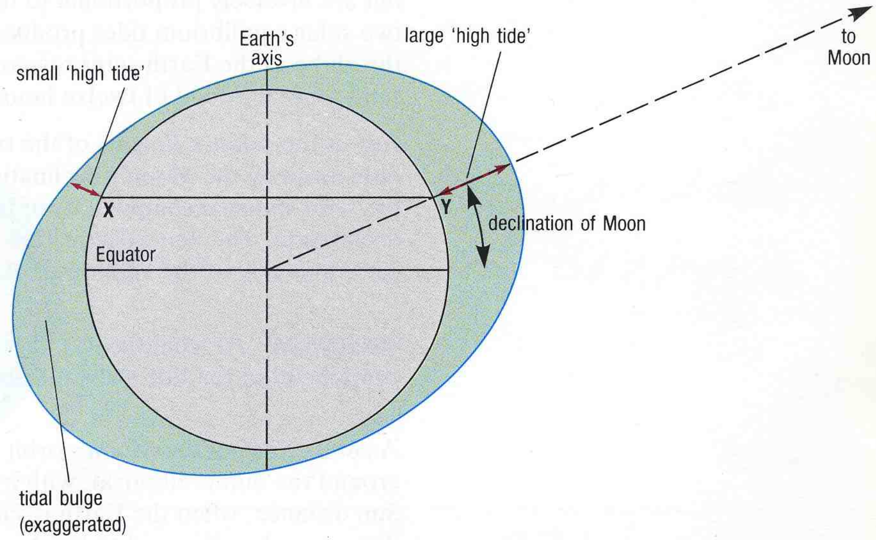

Figure 2.5 The production of unequal tides (tropic tides—see text) at mid-latitudes due to the Moon's declination. An observer at Y would experience a higher high tide than would an observer at X. 12 hours and 25 minutes later their positions would be reversed, i.e. each observer would notice a diurnal inequality.

QUESTION 2.3 What will be the extent of the diurnal tidal inequality due to the Moon as seen by an observer on the Tropic of Capricorn (and within sight of the sea) roughly seven days after the situation shown in Figure 2.5?

Your answer to Question 2.3 emphasizes the cyclical nature of this variation. At maximum declination of 28° the Moon is (roughly) over one of the 'Tropics' (latitude 23°), the diurnal variation is greatest, and the tides are known as **tropic tides**; whereas at minimum (zero) declination (when the Moon is vertically above the Equator), there is no diurnal variation and the tides are called **equatorial tides**.

2 The Moon's elliptical orbit

The orbit of the Moon around the Earth–Moon centre of mass is not circular but elliptical. The consequent variation in distance from Earth to Moon results in corresponding variations in the tide-producing forces. When the Moon is closest to Earth, it is said to be in **perigee**, and the Moon's tide-producing force is increased by up to 20% above the average value. When the Moon is furthest from Earth, it is said to be in **apogee**, and the tide-producing force is reduced to about 20% below the average value. The interval between successive perigees is 27.5 days.

2.2 TIDE-PRODUCING FORCES—THE EARTH–SUN SYSTEM

The Sun also plays its part as a tide-raising agent. Just as the Moon does, the Sun produces tractive forces and two equilibrium tidal bulges. The magnitude of the Sun's tide-producing force is about 0.46 that of the Moon, because, although enormously greater in mass than the Moon, the Sun is some 360 times further from the Earth. As we saw in Section 2.1, tide-producing forces vary directly with the mass of the attracting body, but are inversely proportional to the cube of its distance from Earth. The two solar equilibrium tides produced by the Sun sweep westwards around the globe as the Earth spins towards the east. The solar tide thus has a semi-diurnal period of twelve hours.

Just as the relative heights of the two semi-diurnal lunar tides are influenced by the Moon's declination, so there are diurnal inequalities in the solar-induced components of the tides because of the Sun's declination. The Sun's declination varies over a yearly cycle, and ranges 23° either side of the equatorial plane.

QUESTION 2.4 At what time(s) of the year will the solar-induced component of the tide show maximum diurnal inequality?

As in the case of the Moon's orbit round the Earth, the orbit of the Earth around the Sun is elliptical, with a consequent minimum Earth–Sun distance, when the Earth is said to be at **perihelion**, and a maximum distance, when it is said to be at **aphelion**. However, the difference in distance between perihelion and aphelion is only about 4%, compared with an approximate 13% difference between perigee and apogee.

2.2.1 INTERACTION OF SOLAR AND LUNAR TIDES

In order to understand the interaction between solar and lunar tides, it is helpful to consider the simplest case, where the declinations of the Sun and Moon are both zero. Figure 2.6 shows these conditions, looking down on the Earth from above the North Pole. The direction of rotation of the Earth is shown arrowed, and the solar and lunar tides are shown diagrammatically. The complete cycle of events takes 29.5 days.

In Figure 2.6(a) and 2.6(c), the tide-generating forces of the Sun and Moon are acting in the same directions, and the solar and lunar equilibrium tides coincide. The tidal range produced is large, i.e. the high tide is higher, and the low tide is lower, than the average. Such tides are known as **spring tides**. When spring tides occur, the Sun and Moon are said to be either in **conjunction** (at new Moon—Figure 2.6(a)) or in **opposition** (at full Moon—Figure 2.6(c)). There is a collective term for both situations: the Moon is said to be in **syzygy** (pronounced 'sizzijee').

In Figure 2.6(b) and (d), the Sun and Moon act at right angles to each other, and the solar and lunar tides are out of phase. The tidal range is correspondingly smaller than average. These tides are known as **neap tides**, and the Moon is said to be in **quadrature** when neap tides occur. Inshore fishermen often refer to spring and neap tides by the descriptive names of 'long' and 'short' tides respectively.

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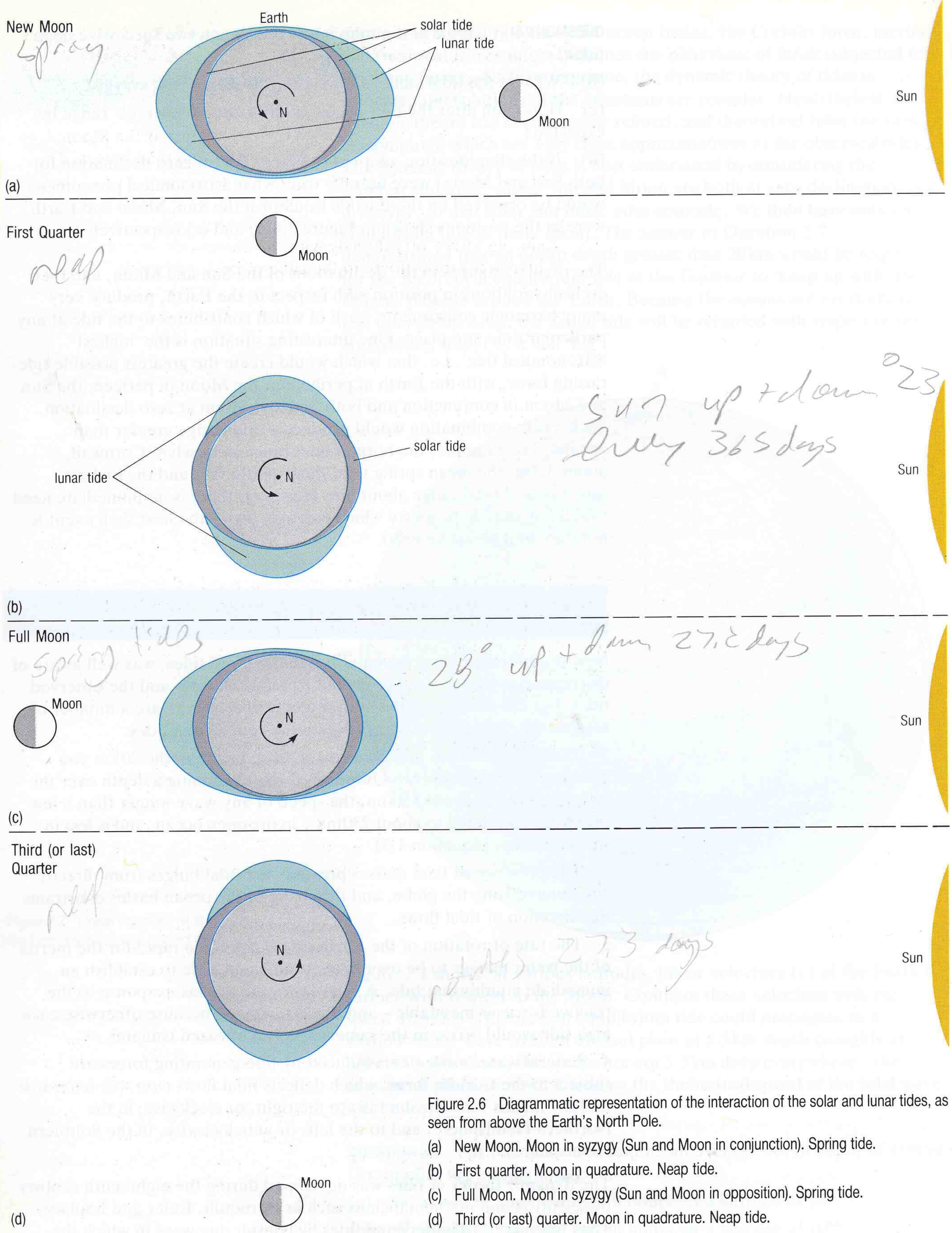


Figure 2.6 Diagrammatic representation of the interaction of the solar and lunar tides, as seen from above the Earth's North Pole.

(a) New Moon. Moon in syzygy (Sun and Moon in conjunction). Spring tide.

(b) First quarter. Moon in quadrature. Neap tide.

(c) Full Moon. Moon in syzygy (Sun and Moon in opposition). Spring tide.

(d) Third (or last) quarter. Moon in quadrature. Neap tide.

QUESTION 2.5(a) What is the time interval between two successive neap tides? 4

(b) What is the tidal state 22 days after the Moon is in syzygy? neap

(c) How soon after the new Moon might a tide of 'average' range be expected? 7 days

(d) If the simplification adopted in Figure 2.6 (of zero declination for both Sun and Moon) were literally true, what astronomical phenomena would be observed on the Earth's Equator if the Sun, Moon and Earth were in the positions shown in Figure 2.6(a) and (c) respectively?

The regular changes in the declinations of the Sun and Moon, and their cyclical variations in position with respect to the Earth, produce very many harmonic constituents, each of which contributes to the tide at any particular time and place. One interesting situation is the 'highest astronomical tide', i.e. that which would create the greatest possible tide-raising force, with the Earth at perihelion, the Moon in perigee, the Sun and Moon in conjunction and both Sun and Moon at zero declination. Such a rare combination would produce a tidal range greater than normal. For example, the normal tidal range at Newlyn, Cornwall, is about 3.5m, the mean spring tidal range about 5m, and the highest astronomical tidal range about 6m. However, there is no immediate need to sell any seaside property which you may own—the next such event is not due until about AD 6580.

2.3 THE DYNAMIC THEORY OF TIDES

Newton, in formulating the equilibrium theory of tides, was well aware of discrepancies between the predicted equilibrium tides and the observed tides, but did not pursue the matter any further. There are a number of reasons why actual tides do not behave as equilibrium tides:

1 The average depth of the oceans is much less than the 20km you calculated as the answer to Question 2.2(b). Assuming a depth over the abyssal plains of about 5500m, the speed of any wave longer than a few kilometres is limited to about 230m$\,$s^{-1} in the open ocean, and is less in shallower seas (equation 1.5).

2 The presence of land masses prevents the tidal bulges from directly circumnavigating the globe, and the shape of the ocean basins constrains the direction of tidal flows.

3 The rate of rotation of the Earth on its axis is too rapid for the inertia of the water masses to be overcome in sufficient time to establish an immediate equilibrium tide. A time-lag in the oceans' response to the tractive forces is inevitable—and this is fortunate because otherwise each high tide would arrive in the same way as an outsized tsunami.

4 Lateral water movements induced by tide-generating forces are subject to the **Coriolis force**, which deflects tidal flows *cum sole* (*cum sole* literally means *with the Sun*, i.e. to the right, or clockwise, in the Northern Hemisphere, and to the left, or anticlockwise, in the Southern Hemisphere).

The **dynamic theory of tides** was developed during the eighteenth century by scientists and mathematicians such as Bernoulli, Euler and Laplace. They attempted to understand tides by considering ways in which the depths and configurations of the ocean basins, the Coriolis force, inertia, and frictional forces might influence the behaviour of fluids subjected to rhythmic forces. As a consequence, the dynamic theory of tides is intricate, and solutions of the equations are complex. Nevertheless, the dynamic theory has been steadily refined, and theoretical tides can now be computed which are very close approximations to the observed tides. The dynamic theory of tides is best understood by considering the simplest situation, where Sun and Moon are both at zero declination, and in syzygy, so that solar and lunar tides coincide. We then have only one equilibrium tide to think about. The answer to Question 2.2 demonstrated that an ocean depth greater than 20km would be required for the theoretical equilibrium tide at the Equator to 'keep up with' the Moon's passage around the Earth. Because the oceans are everywhere less than 20km deep, the actual tide will be retarded with respect to the equilibrium tide, i.e. it will 'lag'.

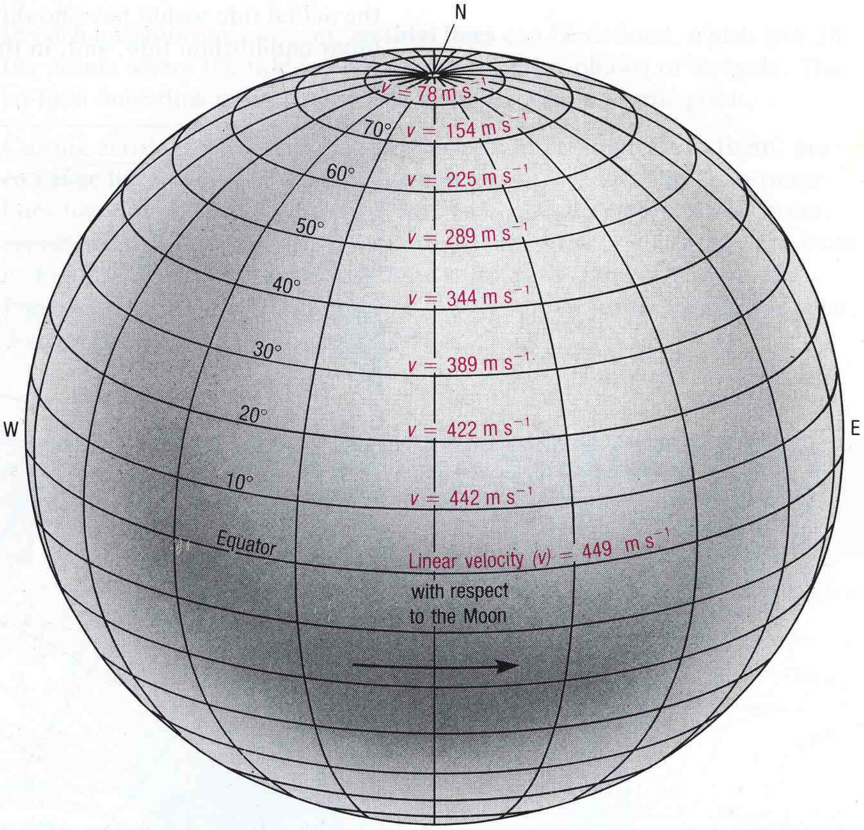


Figure 2.7 Linear velocities (v) at various latitudes of the Earth's surface with respect to the Moon.

Figure 2.7 shows, at various latitudes, linear velocities (v) of the Earth's surface with respect to the Moon. Compare these velocities with the speed at which a theoretical equilibrium tide could propagate as a shallow-water wave over an abyssal plain of 5.5km depth (roughly at 230m s^{-1}). However, the oceans are not 5.5km deep everywhere—the average depth is somewhat less, so the theoretical speed of the tidal wave will be less.

QUESTION 2.6 For this question, assume an average ocean depth of 4080m (to simplify the calculations).

(a) What will be the tidal lag, in hours, at a latitude of 26°?

(b) What will be the tidal lag, in hours, at a latitude of 10°?

From the answer to Question 2.6 it can be seen that at low latitudes the tides will lag 90° of longitude behind the theoretical equilibrium tide. This phenomenon extends either side of the Equator to those latitudes where the Earth's surface (with respect to the Moon) reaches a linear velocity equal to twice the speed at which tides could propagate across the oceans. At these lower latitudes, the lag is limited to 6 hours 12 minutes, so that high tides occur 6 hours 12 minutes and 18 hours 36 minutes after the Moon's passage overhead. Such tides are called **indirect tides**.

At latitudes above about 26°, the tidal lag is less than 6 hours 12 minutes. The precise lag is always constant for any one location, but decreases with increasing latitude, until there is zero lag at about latitude 65°. Consider the speed of the equilibrium tide around the Antarctic Circle (66.5°S). The distance around the circle is some 17300 km, and the linear velocity is thus about 190 m s^{-1}, which is less than the velocity of the tidal wave you assumed when answering Question 2.6(a). Therefore there is no lag, and the actual tide would have no difficulty in keeping up with the theoretical lunar equilibrium tide, and, in theory, high tides would occur at (and 12

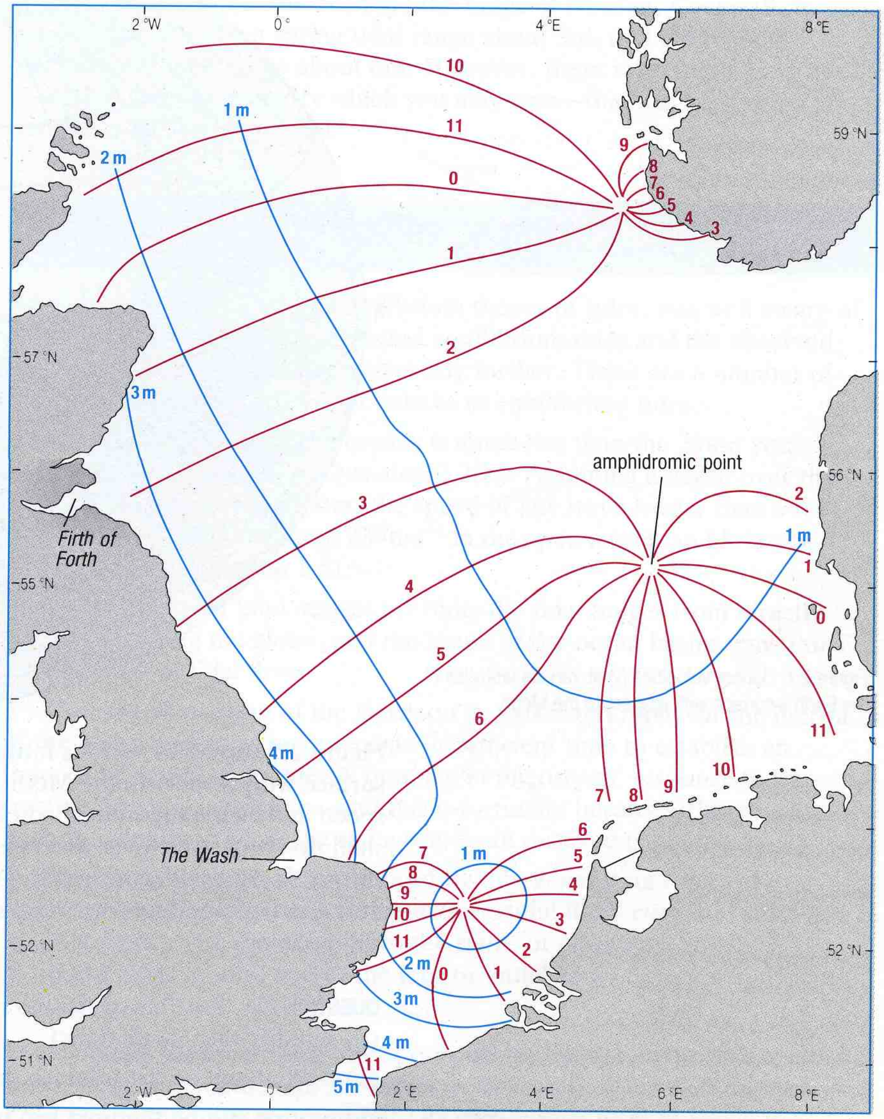


Figure 2.8 Amphidromic systems in the North Sea. The figures on the co-tidal lines indicate the time of high water in 'lunar hours' (i.e. 1/24 of a lunar day of 24.8 hours = about 1 hour and 2 minutes) after the Moon has passed the Greenwich meridian. Blue lines are co-range lines and red lines are co-tidal lines.

hours 25 minutes after) the Moon's passage. Tides of this nature are called **direct tides**. According to the dynamic theory, then, all tides at low latitudes (less than about 26°) would be indirect tides, and all tides at high latitudes (more than about 65°) would be direct. However, tides do not necessarily ebb and flow parallel with the Equator, as the dynamic theory assumes. A considerable longitudinal component of flow occurs, and the actual tidal pattern is thus more complicated than the simple dynamic theory.

So far, we have ignored the configuration of the ocean basins. In fact, the combined constraint of ocean basin geometry and the influence of the Coriolis force results in the development of **amphidromic systems**, in each of which the crest of the tidal wave at high water circulates around an **amphidromic point** once during each tidal period (Figures 2.8 and 2.9). The tidal range is zero at each amphidromic point, and increases outwards away from it.

In each amphidromic system, **co-tidal lines** can be defined, which link all the points where the tide is at the same stage (or phase) of its cycle. The co-tidal lines thus radiate outwards from the amphidromic point.

Cutting across co-tidal lines, approximately at right angles to them, are **co-range lines**, which join places having an equal tidal range. Co-range lines form more or less concentric circles about the amphidromic point, representing larger and larger tidal ranges the further away they are from it. Figure 2.8 shows the amphidromic systems for the North Sea, and Figure 2.9 shows the computed world-wide amphidromic systems for the dominant tidal component (see Section 2.3.1).

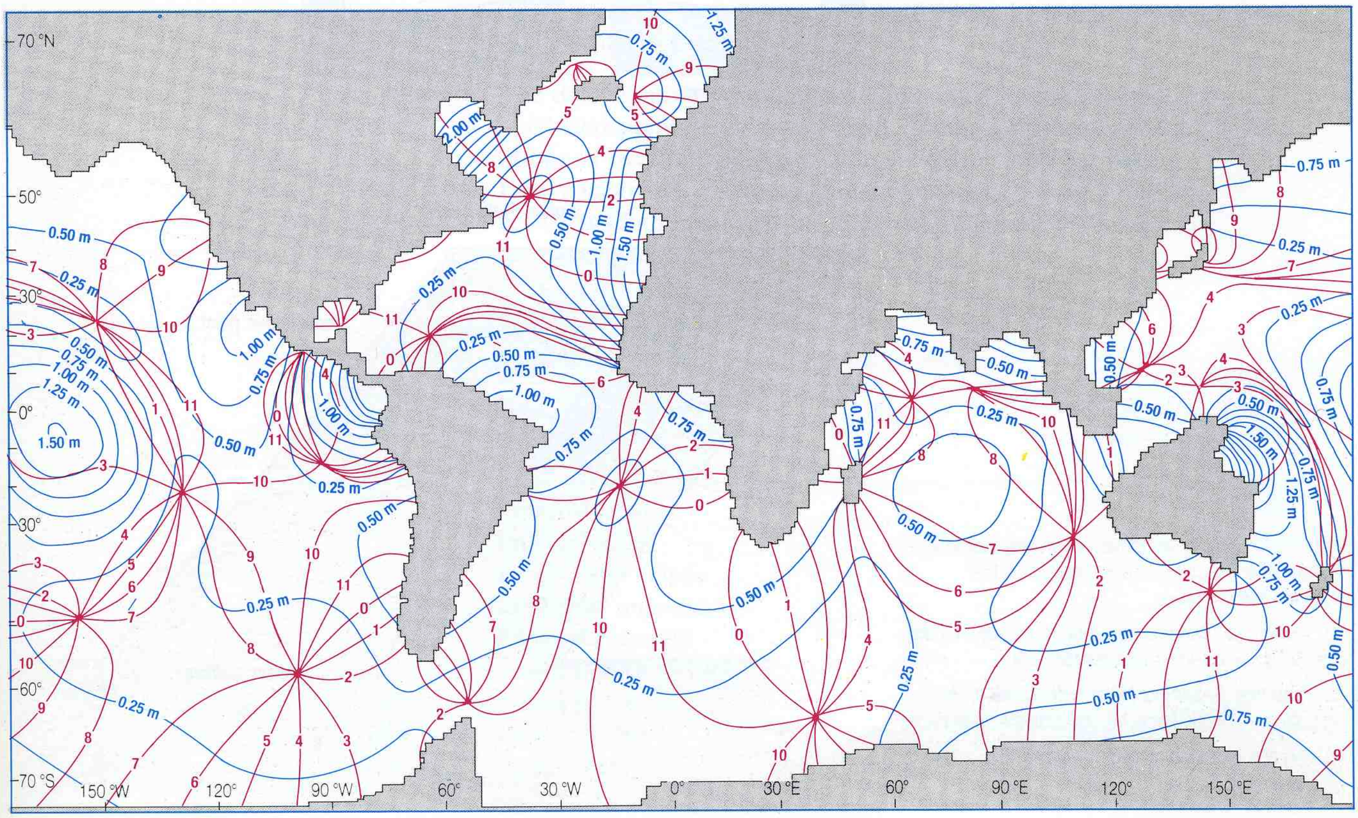


Figure 2.9 Computer-generated diagram of world-wide amphidromic systems for the dominant semi-diurnal lunar tidal component (see Table 2.1). Co-tidal lines are in red and co-range lines are in blue.

QUESTION 2.7 Assume that a high tide coincides with the co-tidal lines marked zero (i.e. '00') on Figure 2.8. At what stage of the tidal cycle is:

(a) The Wash?

(b) The Firth of Forth?

(c) Which one of (a) and (b) has the greater tidal range?

Inspection of Figures 2.8 and 2.9 shows that, with a few exceptions, the tidal waves of amphidromic systems tend to rotate anticlockwise in the Northern Hemisphere and clockwise in the Southern Hemisphere. At first sight, the pattern of rotation appears to conflict with the principle that the Coriolis force deflects moving fluid masses *cum sole*. However, consider the bay shown in Figure 2.10.

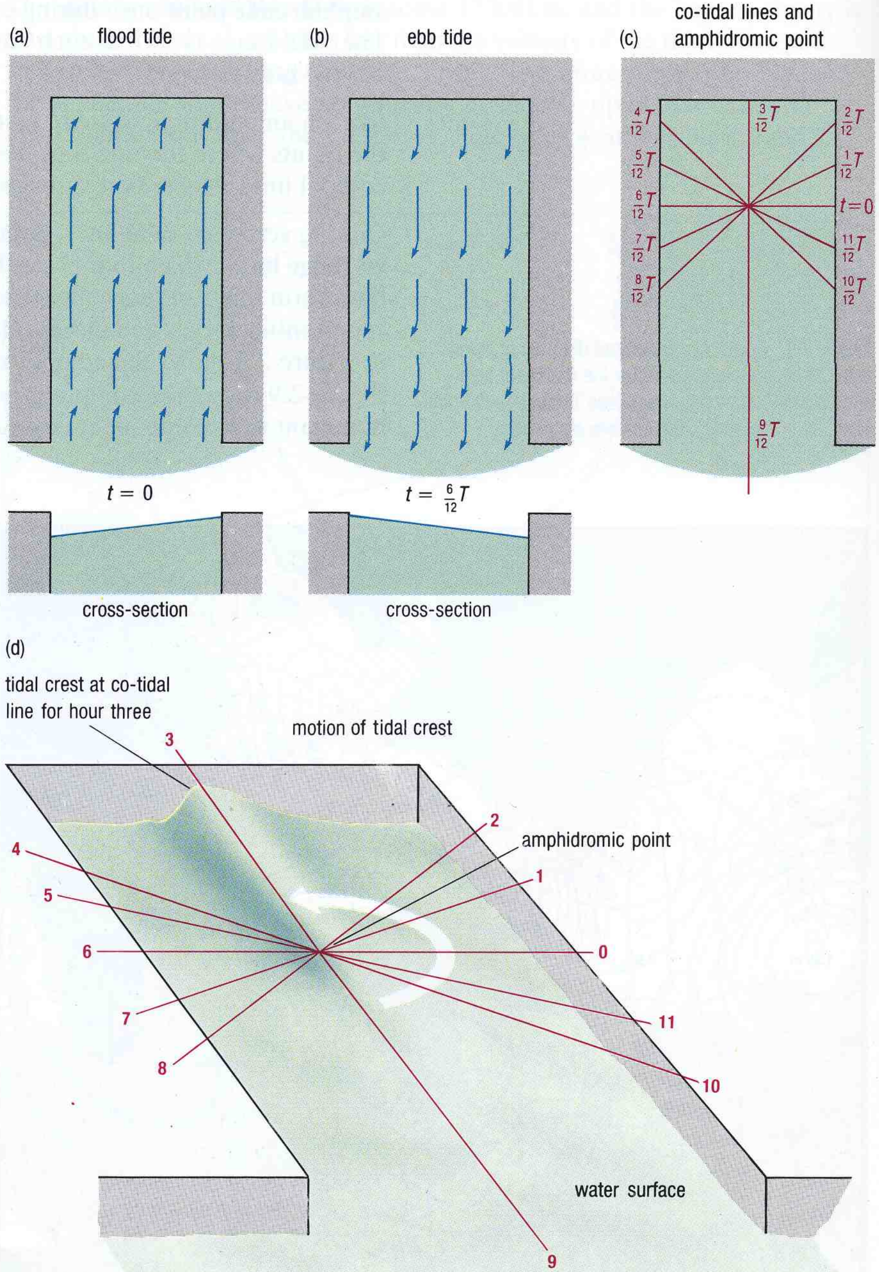


Figure 2.10 The development of an amphidromic system. The hypothetical bay shown is in the Northern Hemisphere.

(a) Flood tide. Water is deflected to the right by the Coriolis force, i.e. towards the east bank.

(b) Ebb tide. Returning water is deflected to the right by the Coriolis force, i.e. towards the west bank.

(c) An amphidromic system is established.

(d) The tidal wave travels anticlockwise.

In Figure 2.10(a) the flooding tide is deflected to the right by the Coriolis force (the bay is in the Northern Hemisphere), and the water is piled up on the eastern side of the bay. Conversely, when the tide ebbs, the water becomes piled up on the western side (Figure 2.10(b)). Hence, because the tidal wave is constrained by land masses, an anticlockwise amphidromic system is set up (Figure 2.10(c) and (d)). It is noteworthy that the main exceptions to the general pattern of rotation shown on Figure 2.9 are systems which are not constrained by land masses (e.g. the South Atlantic, Mid-Pacific and North Pacific amphidromic systems) or where the system rotates about an island (e.g. Madagascar, Ceylon and New Zealand).

Confined amphidromic systems are a type of **Kelvin wave**, in which the restoring force (gravity) is reinforced near the coasts by forces generated by the head of water associated with the sloping sea-surface (Figure 2.10(a) and (b)). Kelvin waves occur where the deflection caused by the Coriolis force is either constrained (as at coasts) or is zero (as at the Equator). Because the Coriolis force acts *cum sole*, Kelvin waves can only travel eastwards at the Equator.

2.3.1 PREDICTION OF TIDES BY THE HARMONIC METHOD

The harmonic method is the most usual and satisfactory method for the prediction of tidal heights. It makes use of the knowledge that the observed tide is the sum of a number of components or **partial tides**, each of whose periods precisely corresponds with the period of one of the relative astronomical motions between Earth, Sun and Moon. Each of the partial tides has an amplitude and phase which is unique to a given location. In this context, phase means the fraction of tidal cycle that has been completed at a given reference time. It depends upon the period of the tide-raising force concerned, and upon the lag of the partial tide for that particular location.

The determination of amplitude and phase for each partial tide at a particular point, such as a seaport, requires a record of tidal heights obtained over a time that is long compared with the periods of the partial tides concerned. As many as 390 components have been identified. Table 2.1 shows four semi-diurnal, three diurnal and two longer-period components.

Table 2.1 Some principal tidal components.

Name of tidal component	Symbol	Period in solar hours	Coefficient ratio ($M_2 = 100$)
Principal lunar	M_2	12.42	100
Principal solar	S_2	12.00	46.6
Larger lunar elliptic	N_2	12.66	19.2
Luni–solar semi-diurnal	K_2	11.97	12.7
Luni–solar diurnal	K_1	23.93	58.4
Principal lunar diurnal	O_1	25.82	41.5
Principal solar diurnal	P_1	24.07	19.4
Lunar fortnightly	M_f	327.86	17.2
Lunar monthly	M_m	661.30	9.1

Even using these few major components, the production of a tide-table for a port for an entire year used to be a very time-consuming activity. In the early years of harmonic analysis, they were computed by hand. The first machine to do the job was invented by Lord Kelvin in 1872. Electronic computers are admirably suited to this repetitive procedure, and tide-tables produced for ports all over the world now take little time to prepare.

The precision achieved by radar altimeters (Section 1.6.1) is such that tidal ranges in the deep oceans can be measured. Information on tidal amplitude and phase was extracted from the *Seasat* data (collected in 1978) by oceanographers working at the Institute of Oceanographic Sciences, Bidston, and found to be in good agreement with predicted values.

2.4 TYPES OF TIDE

Having examined the theory, let us see how actual tides behave in different places and how the different types are classified. The simplest way of classifying tides is by the dominant period of the observed tide. This is based on the ratio (F) of the sum of the amplitudes of the two main diurnal components (K_1 and O_1) to the sum of the amplitudes of the two main semi-diurnal components (M_2 and S_2).

QUESTION 2.8(a) From Figure 2.11, what are the main differences between tidal cycles characterized by high and low values of the ratio F?

(b) Would you expect the interval between spring tides to be 14.75 days at all times, and at all locations, irrespective of the other types of tidal fluctuation?

A high value of F (say above 3.0) implies a diurnal tidal cycle, i.e. only one high tide occurs daily, and fluctuations in the tidal range are largely due to changes in the Moon's declination. Tides are very small at times of zero lunar declination. Low values of F (say less than 0.25) imply a semi-diurnal tide, and the main fluctuations in tidal range are due to the relative positions of Sun and Moon, giving a spring–neap variation.

Between the two extremes are the mixed tidal types, where the daily inequality caused by the declination of the Moon (see Section 2.1.1(cycle 1)) is important, and variations in the amplitudes of, and time intervals between, successive high tides can be considerable. The middle two tidal records on Figure 2.11 show diurnal inequalities where typical 'large tides' alternate with 'half-tides'. The time intervals between successive high waters are unequal when the half-tide is intermittent, as at Manila. Note the change from tropic tides at days 0–6 to equatorial tides at days 7–12 in the cases of both Manila and San Francisco (see Section 2.1.1).

Local effects can modify these basic patterns, particularly the local effects of harmonics (i.e. simple multiples of the frequency) of the partial tides. For example, the quarter-diurnal component M_4 (twice the frequency of M_2, the semi-diurnal or principal lunar component) and the one-sixth-diurnal component M_6 (three times the frequency of M_2) are generated in addition to the semi-diurnal component. In most locations, the effect of the two harmonics is insignificant compared with the principal

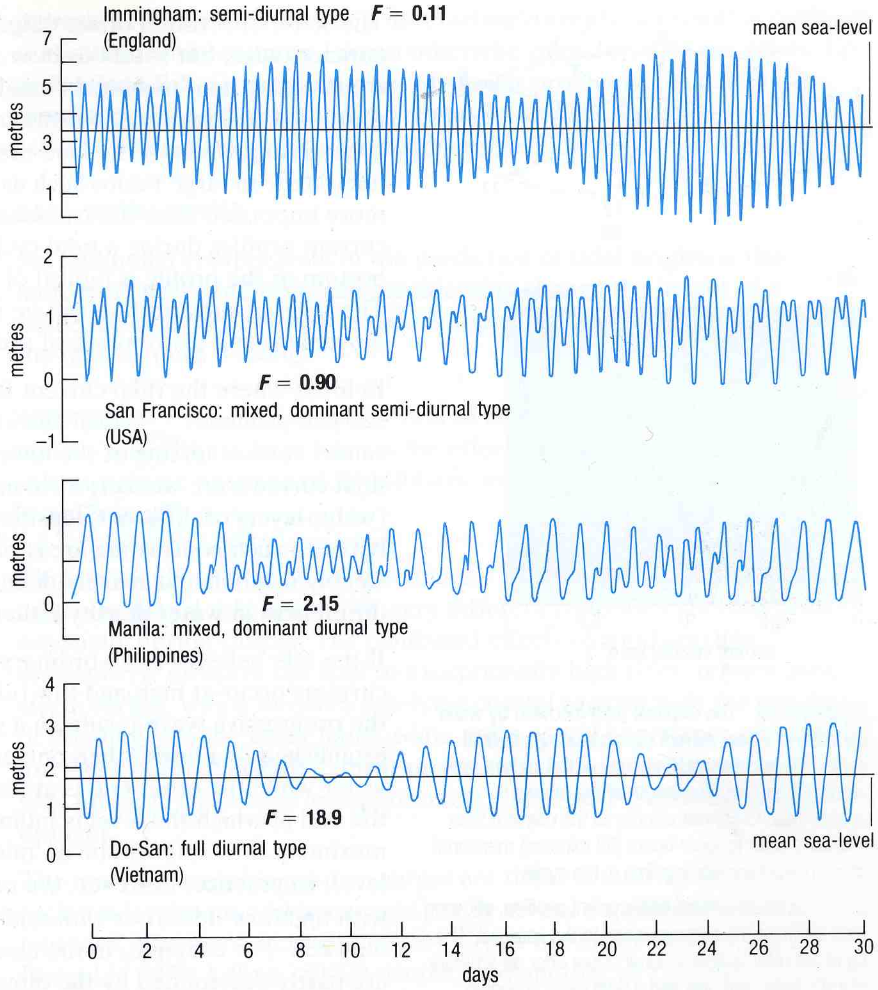
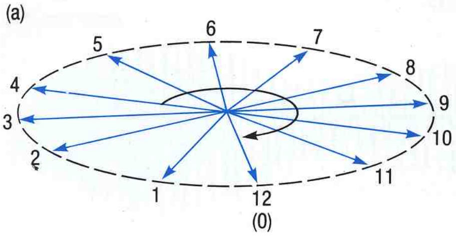


Figure 2.11 Examples of different types of tide curve, in England, the USA, the Philippines, and Vietnam. For full explanation, see text.

component, but along the Dorset and Hampshire coasts of the English Channel each has a larger amplitude than usual. Moreover, the two harmonics are in phase, and their combined amplitude is significant when compared to that of M_2. (Just west of the Isle of Wight, M_2 is about 0.5m, M_4 about 0.15m, and M_6 about 0.2m). The additive effect of all three components largely contributes to the double high waters at Southampton and the double low waters at Portland. There is no truth in the popular myth that double high water at Southampton is caused by the tide flooding at different times around either end of the Isle of Wight.

2.4.1 TIDES AND TIDAL CURRENTS IN SHALLOW SEAS

The vertical water movements associated with the rise and fall of the tide are accompanied by horizontal water motions termed tidal currents. These tidal currents have the same periodicities as the vertical oscillations, but tend to follow an elliptical path and do not normally involve a simple to-and-fro motion (Figure 2.12(a)).



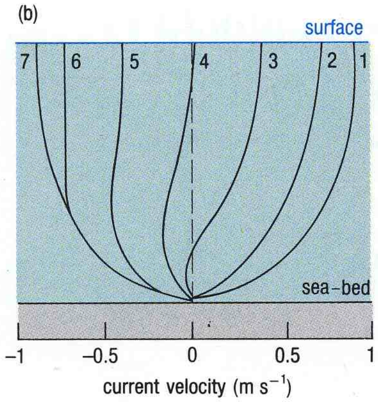


Figure 2.12(a) The elliptical path followed by water particles in a tidal current during a complete tidal cycle. The successive directions of the current are shown by arrows. The length of the arrows is proportional to current velocity at the relevant time. Numbers refer to lunar hours (62 minutes) measured after an arbitrary starting time in the cycle.

(b) A series of vertical tidal current profiles, showing retardation of the currents close to the sea-bed. The numbers refer to time in lunar hours after an arbitrary starting time, and only half a tidal cycle is shown.

The sense of rotation of the ellipse may be either clockwise or anticlockwise, but rotations *cum sole* tend to be favoured if there are no constraining land masses. In small embayments, the effect of the Earth's rotation is insignificant, but the frictional effect of the sea-bed and the constraining effect of the land masses upon the current cannot be neglected. In large basins such as the North Sea, the rotational effect is more important than the frictional effect. Figure 2.12(b) shows a series of current profiles during a tidal cycle. The retardation of the flow near the bottom of the profile is typical of tidal current behaviour in shallow seas. The form of such profiles can be important when considering the movement and distribution of sediments.

In areas where the tidal current is strong enough, the frictional drag at the sea-bed produces a vertical current shear, and the resultant turbulence causes vertical mixing of the lower water layers. In other areas, where tidal currents are weaker, little mixing occurs, and thus stratification (water layers of different densities) can develop. The boundaries (fronts) between such contrasting areas of mixed and of stratified waters are often steeply inclined and sharply defined, so that there are marked lateral differences in water density either side of the front.

If the tide behaves as an ordinary progressive wave, then maximum currents occur at high and low tides. On the other hand, if reflection of the progressive wave results in a standing wave (see Section 1.5.4) being established in a basin, then one end of the basin will experience high water while the other end is at low water. The flow will be directed from the end at which the level is falling to the end at which it is rising, and the maximum flow rate will be at 'mid-tide', when both ends are at the same level. In practice, however, the maximum currents will coincide neither with high/low tides, nor with mid-tides, but will be somewhere in between. For example, in the case of the North Sea, the tidal oscillations are partly determined by the dimensions of the North Sea basin (which has a natural period of about 40 hours) and partly by the progressive semi-diurnal tides entering from the Atlantic. As a result, a standing wave with three nodes tends to develop in the North Sea. The water is deflected by the Coriolis force, and forms three amphidromic systems (as shown in Figure 2.8) by the same mechanism that was outlined in Section 2.3 and Figure 2.10.

In the case of the Bay of Fundy, Nova Scotia, the natural period of oscillation is about 12.5 hours, i.e. it is close to that of the semi-diurnal tide. As a result, there is a strong resonant oscillation, a tidal range of some 15m at the head of the bay, and a strong tidal current at mid-tide.

QUESTION 2.9 The Bay of Fundy is about 270km long, and has an average depth of about 60m. Are these dimensions consistent with the resonant period given in the last paragraph?

Strong tidal currents can also be produced by local constrictions, such as in narrow straits between two seas (e.g. the Straits of Dover). Such currents are known as hydraulic currents and result from the hydraulic pressure gradients caused by differences in sea-level at either end of the straits. The relatively steep gradient of the water surface may result in a tidal race of several knots. Where the French coast reaches out towards the Channel Island of Alderney, spring tides can routinely generate currents of 10 knots ($5.14 m s^{-1}$). Interacting currents can result in confused seas—even white-capping—on an otherwise calm day. The sea off the tip of Portland Bill, Dorset, can present similar problems. Such areas are marked on navigational charts as overfalls, and are particularly to be avoided when waves are steepened by opposition to such tidal currents.

2.4.2 STORM SURGES

An additional complication in the prediction of tidal heights is that meteorological conditions can considerably change the height of a particular tide, and the time at which it occurs. The wind can hold back the tide, or push it along.

QUESTION 2.10 If a head of water 10m in height exerts a pressure of 1 atmosphere (1 bar), then what is the effect on the local sea-level of a fall in atmospheric pressure of 50 millibars, as might occur when a severe storm passes?

Thus, not only wind changes but changes in atmospheric pressure can cause the actual tide level to be very different from the expected value, especially during storms. The combined effects of wind and low atmospheric pressure can lead to exceptionally high tides, termed **positive storm surges**, which threaten low-lying coastal regions with the prospect of flooding. On the other hand, some areas would experience abnormally low tides, termed **negative storm surges**, which cause problems in shallow seas for large ships such as supertankers which have a relatively deep draught.

The most catastrophic positive surges are those caused by tropical cyclones (typhoons and hurricanes) or by severe depressions in temperate latitudes. The worst in recent history struck the north coast of the Bay of Bengal in 1970, killing 250000 people; and another in 1985 caused the loss of 20000 lives. The well-documented North Sea storm surge of 1953 led to local sea-levels up to 3m above normal and caused 1800 deaths in Holland and 300 in England. In this case (as with most positive surges), high spring tides, strong onshore winds and very low barometric pressure combined to produce an abnormal rise in local sea-level. In 1986, more than 30 years after this disaster, a barrier 8km long was built across the eastern Scheldt, completing the final stage of the Delta project which is intended to protect the Netherlands from another flood catastrophe. The completion of the Thames Barrage has provided similar protection for the low-lying areas in and around London. Early warning of storm surges can be given if accurate meteorological and tidal data are available. Forecasting can be aided by satellite tracking of storms, and by computer-modelling of past surges.

Storm surges in the North Sea can, in theory, add up to an extra 4m to the normal tidal height, but fortunately most storm surges (of which there are, on average, about five per year) are of the order of 0.5 to 1m. They are usually associated with eastward-moving depressions, and follow a three-phase pattern:

1 The first signs are evident as a relatively small positive storm surge in the North Atlantic, whereby water is displaced by south-westerly winds to the north-east Atlantic.

2 At the same time as the events in (1), a negative surge is experienced on the east coast of Britain as the south-west winds displace water to the north-east corner of the North Sea. This negative surge travels southwards down the east coast and swings eastward across the southern part of the North Sea, following the amphidromic system shown in Figure 2.8.

3 As the depression moves across Britain and out over the North Sea, the wind veers (i.e. swings in a clockwise direction) to blow from the north-west. The next high tide, by now travelling southwards down the North Sea, is thus reinforced not only by the wind but by the Atlantic surge referred to in (1) above, which by this time is displacing water into the northern part of the North Sea. This large positive surge travels down the east coast of Britain, and reaches a maximum in the south-western corner of the North Sea. The problem is compounded, not only by the funnelling effect imposed by the basin shape, but by the fact that the arrival of the surge may coincide with the arrival of the low pressure area in the centre of the depression, thus increasing local sea-level still further.

2.4.3 TIDES IN RIVERS AND ESTUARIES

Many of the world's rivers are in part tidal, because sea-levels have risen since the last glacial period. In such cases, the lower river valleys have become inundated by the sea, forming estuaries or rias. The tides thus propagate up the estuaries, and in some cases into the lower reaches of the rivers. The distinction between an estuary and the tidal reaches of a river is somewhat blurred, and for the purposes of this Section the two are treated as one. The speed of tidal propagation into an estuary depends upon the water depth. Hence the wave crest (high water) will travel faster than the wave trough (low water). As a result, there is an asymmetry in the tidal cycle, with a relatively long time interval between high water and the succeeding low water, and a shorter interval between low water and the next high tide (Figure 2.13).

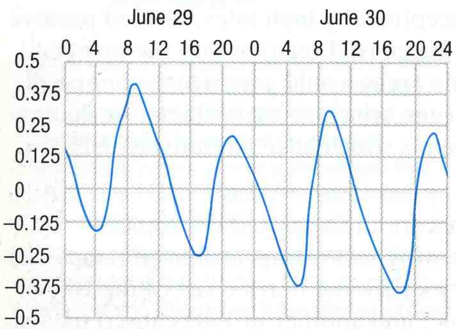


Figure 2.13 A tidal curve for the Hudson River estuary near Albany, New York, showing a typical river tide with peaks tending to catch up with the preceding trough. Numbers on the horizontal axis are time in hours.

The maximum speeds of the tidal currents associated with estuarine tides may not always be in phase with the tidal crests and troughs. Thus, at the estuary mouth, the maximum speed of the flooding tide may coincide with high water, yet further up-river high tide may well occur at the same time as slack water (i.e. zero current). However, the ebb current will invariably persist for longer than the flood, partly as a result of the asymmetry of the tidal cycle already referred to, and partly because the freshwater discharge into the river results in a net seaward discharge of water. Many towns and cities sited near such estuaries rely upon this net seaward flow to carry away sewage, a strategy which is sometimes only a 'mixed' success.

In some tidal rivers, where either the river channel narrows markedly, or the gradient of the river bed steepens, a **tidal bore** may develop. The formation of tidal bores has features in common with the propagation of waves against a counter-current (see Section 1.5.1). The rising tide may force the tidal wave-front to move faster than a shallow-water wave can freely propagate into water of that depth (*cf.* equation 1.5). When this happens, a shock wave is formed, which moves upstream as a rolling wall of water, or tidal bore. It is analogous to the 'sonic boom' that occurs when a pressure disturbance is forced to travel faster than the speed of sound. Most tidal bores are relatively small, of the order of 0.5m high, but some can be up to ten times that height. The Severn River bore in England is some 1–2 m high, whereas the Amazon bore (called the *pororoca*) reaches about 5 m, and moves upstream at about 12 knots. Other rivers where bores develop include the Colorado, Trent, Elbe, Yangtze and the Petitcodiac, which flows into the Bay of Fundy, notable for its large tidal range (Section 2.4.2).

2.4.4 TIDAL POWER

Power can be generated by holding incoming and outgoing tides behind a dam, using the head of water so produced to drive turbines for electricity generation. The tidal range controls the potential energy available at any locality, and must exceed 5 m for electricity generation to be economic. Suitable localities are limited to those where such a range exists and where dams can be built practicably (Figure 2.14).

(a)

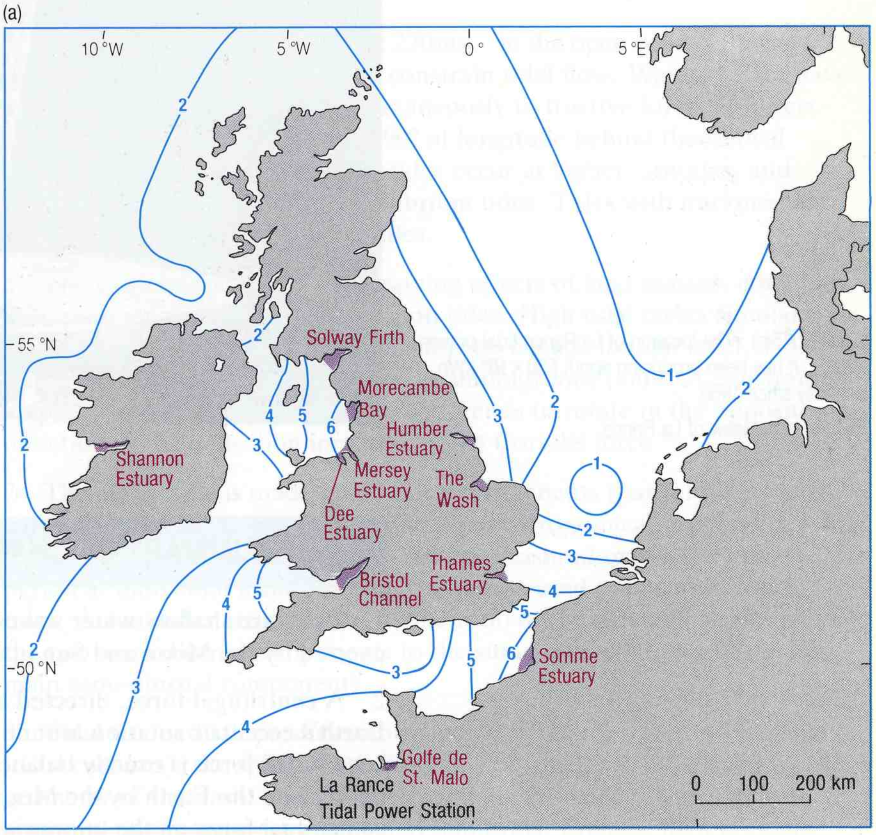
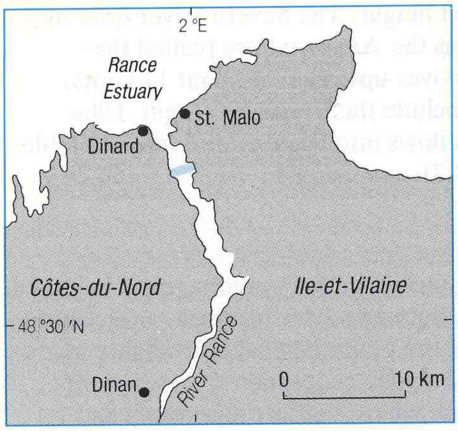


Figure 2.14 The ranges of the M_2 tide (in metres), and the sites of actual (La Rance) and potential tidal-power barrages.

One such site is the Rance estuary in Brittany (Figure 2.15(a) and (b)), which has been in use since 1966. A much larger scheme for Britain's Severn estuary has been proposed and discussed many times. Although such a scheme would produce an appreciable proportion (in the order of 6%) of Britain's electrical power requirements, dam construction would affect the patterns of currents and sediment movements, while ecological disturbance would be inevitable—all factors to be considered whenever schemes of this kind are planned.



(a)


(b)

Figure 2.15(a) The location of La Rance tidal power station. It has been producing about 550×10^6 kWh annually since 1966.

(b) An aerial view of La Rance.

2.5 SUMMARY OF CHAPTER 2

1 Tides are shallow-water waves, generated by gravitational forces exerted by the Moon and Sun upon the oceans.

2 A centrifugal force, directed away from the Moon, results from the Earth's eccentric rotation around the Earth–Moon centre of mass. This centrifugal force is exactly balanced *in total* by the gravitational force exerted on the Earth by the Moon. However, gravitational force exceeds centrifugal force on the 'moonside' of Earth, resulting in tide-producing forces directed towards the Moon, whereas on the other side of the Earth centrifugal force exceeds gravitational force, resulting in tide-producing forces directed away from the Moon.

3 Tractive forces (horizontal components of tide-producing forces) are maximal on two small circles either side of the Earth, and produce two (theoretical) equilibrium tidal bulges—one directed towards the Moon, and the other directed away from it. As the Earth rotates with respect to the Moon (a period of 24 hours 50 minutes), the equilibrium tidal bulges would need to travel in the opposite direction in order to maintain their positions relative to the Moon. In the simplest case, when the Moon is overhead at the Equator, the tidal bulges travel in the same plane as the Equator, and at all points the two bulges cause two equal high tides daily

(equatorial tides). The Moon is not always overhead at the Equator, but has a declination of up to 28° either side of it, so that the plane of travel of the tidal bulges, when offset with respect to the Equator, gives two unequal, or tropic, tides daily. The declination varies over a 27.2 day cycle. The elliptical orbit of the Moon about the Earth causes variation in the tide-producing forces (up to 20% from the mean value).

4 By analogy with the Moon, the Sun produces equilibrium tides which show inequalities related to the Sun's declination (up to 23° either side of the Equator), and vary in magnitude due to the elliptical orbit of the Earth around the Sun. The Sun's tide-raising force has about 46% of the strength of the Moon's. Solar tides interact with lunar tides. When Sun and Moon are in syzygy, the effect is additive, giving large-ranging spring tides; but when Sun and Moon are in quadrature, tidal ranges are small (neap tides). The full cycle, which includes two neaps and two springs, takes 29.5 days.

5 Tidal speed is limited to about 230 $m\,s^{-1}$ in the open oceans (less in shallower seas), and land masses constrain tidal flow. Water masses have inertia, and do not respond instantaneously to tractive forces. Indirect tides, found at low latitudes, lag 90° of longitude behind theoretical equilibrium tides, whereas direct tides occur at higher latitudes, and coincide with the theoretical equilibrium tides. Tides with intermediate characteristics occur at mid-latitudes.

6 The Coriolis force, and constraining effects of land masses, combine to impose amphidromic systems upon tides. High tidal crests circulate around amphidromic points which show no change in tidal level. Tidal range increases with distance from an amphidromic point. A constrained amphidromic system, as in a large bay, tends to rotate in the opposite direction to the deflection imposed by the Coriolis force.

7 The actual tide is made up of many components (partial tides), each corresponding to the period of a particular astronomical motion involving Earth, Sun or Moon. Partial tides can be measured over a long time period at individual locations, and the results used to compute future tides. Actual tides are classified by the ratio of the summed amplitudes of the two main diurnal components to the summed amplitudes of the two main semi-diurnal components.

8 Tidal rise and fall produces corresponding lateral water movements (tidal currents), the speeds and directions of which are influenced by the geometry of the basin and its constraining land masses. Areas of low atmospheric pressure cause elevated sea-levels, whereas high pressure depresses sea-level. A strong wind can hold back a high tide or reinforce it. Storm surges are caused by large changes in atmospheric pressure and the associated strong winds. Positive storm surges often result in catastrophic flooding.

9 In estuaries, the tidal crest travels faster than the tidal trough because speed of propagation depends upon water depth; hence the low-water to high-water interval is shorter than that from high-water to low-water. Tidal bores develop where tides are constrained by narrowing estuaries and the wave-front is forced by the rising tide to travel faster than the depth-determined speed of a shallow-water wave. Where tidal ranges are large and the water can be trapped by dams, the resultant heads of water can be used for hydro-electric power generation.

Now try the following questions to consolidate your understanding of this Chapter.

QUESTION 2.11 Write an expression for the tide-producing force at point P on Figure 2.3, using the terms as defined for equations 2.1, 2.2 and 2.3. It is not essential to try to simplify or approximate the expression.

QUESTION 2.12 Which of the following statements are true?

(a) 'In syzygy' has the same meaning as 'in opposition'.

(b) Neap tides would be experienced during an eclipse of the Sun.

(c) Spring tides do not occur in the autumn.

(d) The lowest sea-levels of the spring–neap cycle occur at low tide while the Moon is in quadrature.

QUESTION 2.13 Briefly summarize the factors accounting for differences between the equilibrium tides and the observed tides.

QUESTION 2.14 How will each of the following influence the tidal range at Immingham (Figure 2.11):

(a) The Earth's progress from perihelion to aphelion?

(b) The occurrence of a tropic tide?

(c) A 30 millibar rise in atmospheric pressure?

(d) A spring tide?

QUESTION 2.15 The caption to Figure 2.1 states that the view is from above one of the poles. Which pole is in view?