CHAPTER 1 WAVES

'...the chidden billow seems to pelt the clouds...'
Othello, Act II, Sc. I.

Sea waves have attracted attention and comment throughout recorded history. Aristotle (384–322 BC) observed the existence of a relationship between wind and waves, and the nature of this relationship has been the subject of study ever since. However, at the present day, understanding of the mechanism of wave formation and the way that waves travel across the oceans is by no means complete. This is partly because observations of wave characteristics at sea are difficult, and partly because mathematical models of wave behaviour are based upon the dynamics of idealized fluids, and ocean waters do not conform precisely with those ideals. Nevertheless, some facts about waves are well established, at least to a first approximation, and the purpose of this Chapter is to outline the qualitative aspects of water waves, and to explore some of the simple relationships of wave dimensions and characteristics.

We start by examining the dimensions of an idealized water wave, and the terminology used to describe them (Figure 1.1).

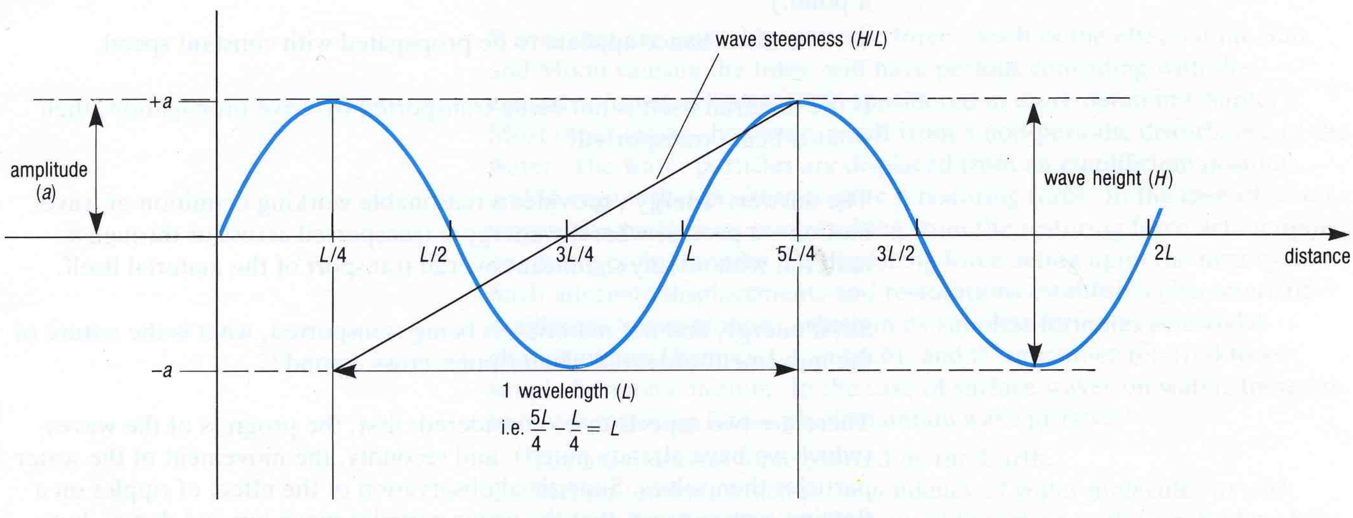


Figure 1.1 Vertical profile of two successive idealized ocean waves, showing their linear dimensions and sinusoidal shape.

Wave height (H) refers to the overall vertical change in height between the wave crest (or peak) and the wave trough. The wave height is twice the wave **amplitude** (a). **Wavelength** (L) is the distance between two successive peaks (or two successive troughs). **Steepness** is defined as wave height divided by wave length (H/L), and, as can be seen in Figure 1.1, is not the same thing as the slope between a wave crest and its adjacent trough.

In addition to having dimensions in space at a fixed instant in time (Figure 1.1), waves have dimensions in time at a fixed point in space. The time interval between two successive peaks (or two successive troughs) passing a fixed point is known as the **period** (T), and is measured in seconds. The number of peaks (or the number of troughs) which pass a fixed point per second is known as the **frequency** (f).

QUESTION 1.1 If a wave has a frequency of $0.2 s^{-1}$, what is its period?

As the answer to Question 1.1 shows, period is the reciprocal of frequency. We will return to this concept in Section 1.2.

1.1 WHAT ARE WAVES?

Waves are a common occurrence in everyday life, and include such examples as sound, the motion of a plucked guitar string, ripples on a pond, and the billows on the ocean. It is not easy to define a wave. Before attempting to do so, let us consider some of the characteristics of wave motion:

1 A wave transfers a disturbance from one part of a material to another. (The disturbance caused by dropping a stone into a pond is transmitted across the pond by ripples.)

2 The disturbance is propagated through the material without any substantial overall motion of the material itself. (A floating cork merely bobs up and down on the ripples, but experiences very little overall movement in the direction of travel of the ripples.)

3 The disturbance is propagated without any significant distortion of the wave form. (A ripple shows very little change in shape as it travels across a pond.)

4 The disturbance appears to be propagated with constant speed.

If the material itself is not being transported by wave propagation, then what *is* being transported?

The answer, 'energy', provides a reasonable working definition of wave motion—a process whereby energy is transported across or through a material without any significant overall transport of the material itself.

So, if energy, and not material, is being transported, what is the nature of the movement observed when ripples cross a pond?

There are two aspects to be considered: first, the progress of the waves (which we have already noted), and secondly, the movement of the water particles themselves. Superficial observation of the effect of ripples on a floating cork suggests that the water particles move 'up and down', but closer observation will reveal that, provided the water is very much deeper than the ripple height, the cork is describing a nearly circular path in a vertical plane, parallel with the direction of wave movement. In a more general sense, the particles are displaced from an equilibrium position, and then return to that position. Thus, the particles experience a displacing force and a restoring force. The nature of these forces is often used in the descriptions of various types of waves.

1.1.1 TYPES OF WAVES

All waves can be regarded as **progressive waves**, in that energy is moving through, or across the surface of, the material.

The so-called **standing wave**, of which the plucked guitar string is an example, can be considered as the sum of two progressive waves of equal dimensions, but travelling in opposite directions. We examine this in more detail in Section 1.5.4.

Waves which travel through the material are called body waves. Examples of body waves are seismic P- and S-waves, and sound waves, but our main concern in this Volume is with **surface waves**. The most familiar surface waves are those which occur at the interface between atmosphere and ocean, but surface waves can occur at the interface between any two bodies of fluid. For example, waves can occur at an interface between two layers of ocean water of differing densities. Because the interface is a surface, such waves are, strictly speaking, surface waves, but oceanographers usually refer to them as **internal waves**. Oscillations are more easily set up at an internal interface than at the sea-surface, because the difference in density between two water layers is smaller than that between water and air. Hence, less energy is required to generate internal waves than surface waves of similar amplitude. Internal waves travel more slowly than surface waves of comparable amplitude, and are of importance in the context of vertical mixing processes in the oceans. Surface waves are caused either by forces resulting from relative motion between two layers of fluid, as for example the wind blowing over the sea, or by an external force that disturbs the fluid. Examples of such external forces range from raindrops falling into a pond, through diving gannets, ocean-going liners and earthquakes, to the gravitational attractions of the Sun and Moon.

Waves that are caused by periodic forces, such as the effect of the Sun and Moon causing the tides, will have periods coinciding with the causative forces. This aspect is considered in more detail in Chapter 2. Most other waves, however, result from a non-periodic disturbance of the water. The water particles are displaced from an equilibrium position, and to regain that position require a restoring force. In the case of water waves, the particle motion resulting from the restoring force acting upon one wave cycle provides the displacing force acting upon the next cycle. Such alternate displacements and restorations establish a characteristic oscillatory 'wave motion', which in its simplest form has sinusoidal characteristics (Figures 1.1 and 1.6), and is sometimes referred to as simple harmonic motion. In the case of surface waves on water, there are two such restoring forces which maintain wave progress:

1 The gravitational force exerted by the Earth.
2 Surface tension, which is the tendency of water molecules to stick together and present the smallest possible surface to the air. So far as the effect on water waves is concerned, it is as if a weak elastic skin were stretched over the water surface.

Water waves are affected by both of these forces. In the case of waves with wavelengths less than about 1.7cm, the principal maintaining force is surface tension, and such waves are known as **capillary waves**. Capillary waves are important in the context of remote sensing of the oceans (Section 1.6.1). However, the main interest of oceanographers lies with surface waves of wavelengths greater than 1.7cm, and the principal maintaining force for such waves is gravity; hence they are known as **gravity waves**. Figure 1.2 summarizes some wave types and their causes.

Not all waves are displaced in a vertical plane. Because atmosphere and oceans are on a rotating Earth, variation of planetary vorticity with latitude causes deflection of atmospheric and oceanic currents, and

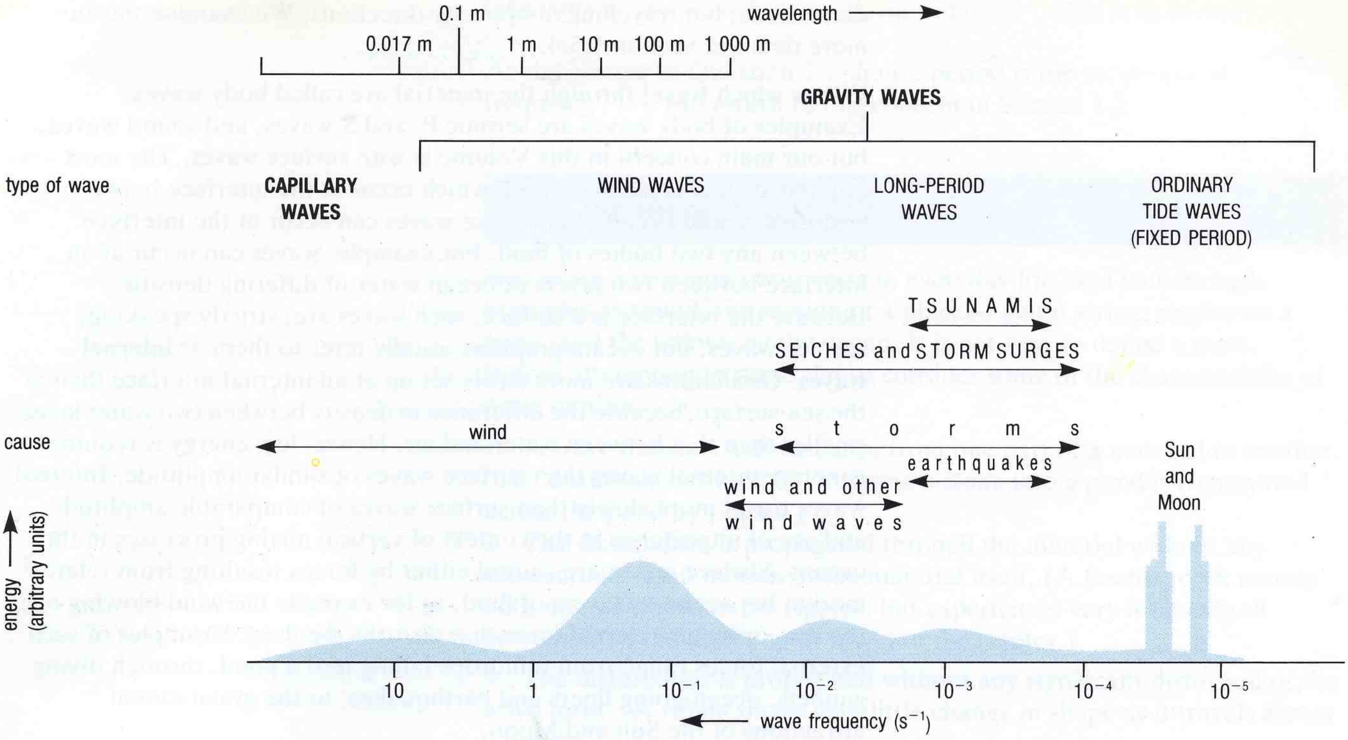


Figure 1.2 Types of surface waves, showing the relationships between wavelength, wave frequency, the nature of the displacing forces, and the relative amounts of energy in each type of wave. Unfamiliar terms will be explained later.

provides restoring forces which establish oscillations in a horizontal plane, so that easterly or westerly currents tend to swing back and forth about an equilibrium latitude. These large-scale waves are known as planetary or **Rossby waves**, and may occur as surface or as internal waves.

1.1.2 WIND-GENERATED WAVES ON THE OCEAN

In 1774, Benjamin Franklin said: 'Air in motion, which is wind, in passing over the smooth surface of the water, may rub, as it were, upon that surface, and raise it into winkles, which, if the wind continues, are the elements of future waves'.

In other words, if two fluid layers having differing speeds are in contact, and there is frictional stress between them, there is a transfer of energy. At the sea-surface, most of the transferred energy results in waves, although a small proportion is manifest as wind-driven currents. In 1925, Harold Jeffreys suggested that waves obtained energy from the wind by virtue of pressure differences caused by the sheltering effect provided by wave crests (Figure 1.3).

Although Jeffreys' hypothesis fails to explain the formation of very small waves, it does seem to work if:

1 Wind speed exceeds wave speed.
2 Wind speed exceeds $1\,\mathrm{m\,s^{-1}}$.
3 The waves are steep enough to provide a sheltering effect.

Empirically, it can be shown that the sheltering effect is at a maximum when wind speed is approximately three times the wave speed. In the open oceans, most wind-generated waves have a steepness (H/L) of about 0.03 to 0.06. In general, the greater the amount by which wind speed

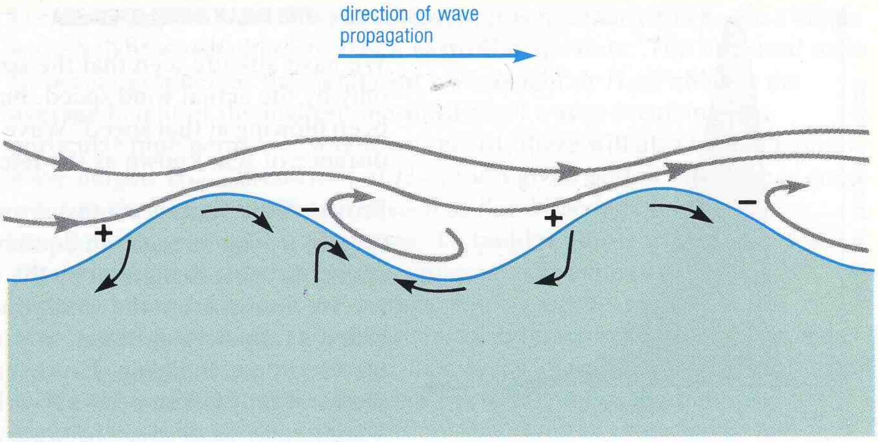


Figure 1.3 Jeffreys' 'sheltering' model of wave generation. Curved lines indicate air flow; short, straight arrows show water movement, which will be explained more fully in Section 1.2.1. The rear face of the wave against which the wind blows experiences a higher pressure than the front face, which is sheltered from the force of the wind. Air eddies are formed in front of each wave, leading to differences in air pressure. The excesses and deficiencies of pressure are shown by plus and minus signs respectively. The pressure difference pushes the wave along.

exceeds wave speed, the steeper the wave. However, as we shall see later, wave speed in deep water is not related to wave steepness, but to wavelength—the greater the wavelength, the faster the wave travels.

QUESTION 1.2 Two waves have the same height, but differing steepness. Which of the two waves will travel the faster?

Consider the sequence of events if, after a period of calm weather, a wind starts to blow, rapidly increases to a gale, and continues to blow at constant gale force for some considerable time. No significant wave growth occurs until the wind speed exceeds $1\,\mathrm{m\,s^{-1}}$. Then, small steep waves form as the wind speed increases. Even after the wind has reached a constant gale force, waves continue to grow with increasing rapidity until they reach a size and wavelength (and hence a speed) which corresponds to one-third of the wind speed. Beyond this point, the waves continue to grow in size, wavelength and speed, but at an ever-diminishing rate. On the face of it, one might expect that wave growth would continue until wave speed was the same as wind speed. However, in practice wave growth ceases whilst wave speed is still at some value below wind speed. This is because:

1 Some of the wind energy is transferred to the ocean surface via a tangential force, and thus produces a surface current.

2 Some wind energy is dissipated by friction.

3 Energy is lost from larger waves as a result of **white-capping**, i.e. breaking of the tip of the wave crest because it is being driven forward by the wind faster than the wave itself is travelling. Much of the energy dissipated during white-capping is converted into forward momentum of the water itself, reinforcing the surface current initiated by process 1 above.

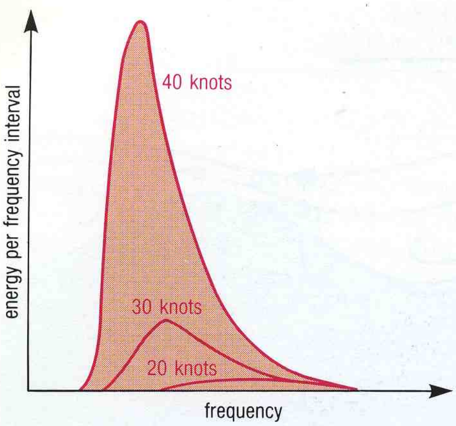


Figure 1.4 Wave energy spectra for three fully developed seas, related to wind speeds of 20, 30 and 40 knots (10.3, 15.45 and 20.6 ms^{-1} respectively). The area under each curve is a measure of the total energy in that particular wave field.

1.1.3 THE FULLY DEVELOPED SEA

We have already seen that the size of waves in deep water is governed not only by the actual wind speed, but also by the length of time the wind has been blowing at that speed. Wave size also depends upon the unobstructed distance of sea, known as the **fetch**, over which the wind blows.

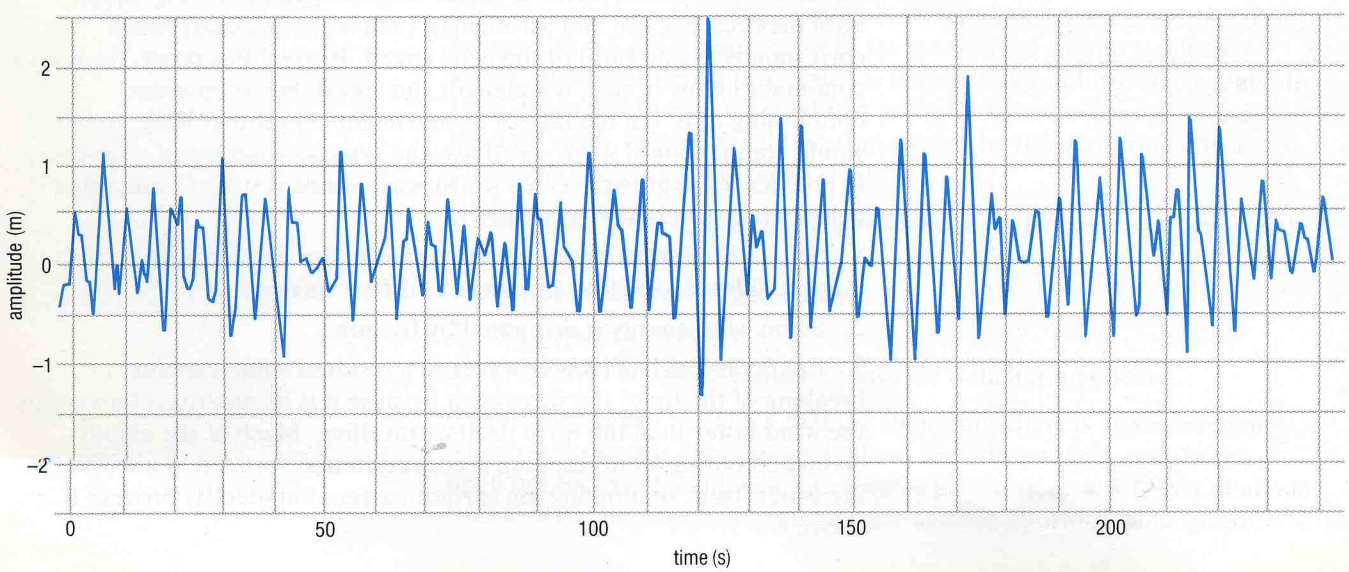
Provided the fetch is extensive enough, and the wind blows at constant speed for long enough, an equilibrium is eventually reached, in which energy is being dissipated by the waves at the same rate as the waves receive energy from the wind. Such an equilibrium results in a sea state called a **fully developed sea**, in which the size and characteristics of the waves are not changing. However, the wind speed is usually variable, so the ideal fully developed sea, with waves of uniform size, rarely occurs. Variation in wind speed produces variation in wave size, so, in practice, a fully developed sea consists of a range of wave sizes known as a **wave field**. Note that a range of wave sizes will also result from waves coming into the area from elsewhere, and from wave–wave interaction—a concept explained in Section 1.4.3. Oceanographers find it convenient to express a wave field as a spectrum of wave energies (Figure 1.4). The energy contained in a wave is proportional to the square of the wave height (see Section 1.4).

QUESTION 1.3 Examine Figure 1.4. Does the energy contained in a wave field increase or decrease as the average frequency of the constituent waves increases?

1.1.4 WAVE HEIGHT AND WAVE STEEPNESS

As was hinted in Section 1.1.3, the height of any particular wave is influenced by many wave components, each of different frequency and amplitude, which move into and out of phase with and across each other. In theory, if the heights and frequencies of all the contributing waves are known, it is possible to predict the heights and frequencies of the largest waves accurately. In practice, this is rarely possible. Figure 1.5 illustrates the range of wave heights which occur over a short time at one location—there is no obvious pattern to the variation of wave height.

Figure 1.5 A typical wave record, i.e. a record of variation in water level with time at one position.



For many applications of wave research, it is necessary to choose a single wave height which characterizes a particular sea state. The one used most by oceanographers is the **significant wave height** or $H_{1/3}$, which is the average height of the highest one-third of all waves occurring in a particular time period. In any wave record, there will also be a maximum wave height, H_{max}. Prediction of H_{max} for a given period of time has great value in the design of structures such as flood barriers, harbour installations, and drilling platforms. To build these structures with too great a margin of safety would be unnecessarily expensive, but to underestimate H_{max} could have tragic consequences. However, it is necessary to emphasize the essentially random nature of H_{max}. The wave $H_{max(25\ years)}$ will occur on average once every 25 years. This does not mean such a wave will automatically occur every 25 years—there may be periods much longer than that without one. On the other hand, two such waves might appear next week.

As wind speed increases, so $H_{1/3}$ in the fully developed sea increases. The relationship between sea state, $H_{1/3}$ and wind speed is expressed by the **Beaufort Scale** (Table 1.1). The Beaufort Scale can be used to estimate wind speed at sea, but is valid only for waves generated within the local weather system, and assumes that there has been sufficient time for a fully developed sea to have become established.

Table 1.1 A selection of information from the Beaufort Wind Scale.

Beaufort No.	Name	Wind speed knots	Wind speed $m\,s^{-1}$	State of the sea-surface	Wave height* (m)
0	Calm	<1	0.0–0.2	Sea like a mirror.	0
1	Light air	1–3	0.3–1.5	Ripples with appearance of scales; no foam crests.	0.1–0.2
2	Light breeze	4–6	1.6–3.3	Small wavelets; crests have glassy appearance but do not break.	0.3–0.5
3	Gentle breeze	7–10	3.4–5.4	Large wavelets; crests begin to break; scattered white horses.	0.6–1.0
4	Moderate breeze	11–16	5.5–7.9	Small waves, becoming longer; fairly frequent white horses.	1.5
5	Fresh breeze	17–21	8.0–10.7	Moderate waves taking longer form; many white horses and chance of some spray.	2.0
6	Strong breeze	22–27	10.8–13.8	Large waves forming; white foam crests extensive everywhere and spray probable.	3.5
7	Moderate gale	28–33	13.9–17.1	Sea heaps up and white foam from breaking waves begins to be blown in streaks; spindrift begins to be seen.	5.0
8	Fresh gale	34–40	17.2–20.7	Moderately high waves of greater length; edges of crests break into sprindrift; foam is blown in well-marked streaks.	7.5
9	Strong gale	41–47	20.8–24.4	High waves; dense streaks of foam; sea begins to roll; spray may affect visibility.	9.5
10	Whole gale	48–55	24.5–28.4	Very high waves with overhanging crests; sea-surface takes on white appearance as foam in great patches is blown in very dense streaks; rolling of sea is heavy and visibility reduced.	12.0
11	Storm	56–64	28.5–32.7	Exceptionally high waves; sea covered with long white patches of foam; small and medium-sized ships might be lost to view behind waves for long times; visibility further reduced.	15.0
12	Hurricane	>64	>32.7	Air filled with foam and spray; sea completely white with driving spray; visibility greatly reduced.	>15

* $H_{1/3}$, i.e. the significant wave height.

The absolute height of a wave is less important to sailors than is its steepness (H/L). As was mentioned in Section 1.1.2, most wind-generated waves have a steepness in the order of 0.03 to 0.06. Waves steeper than this can present problems to shipping, but fortunately it is very rare for wave steepness to exceed 0.1. In general, wave steepness diminishes with increasing wavelength. The short choppy seas rapidly generated by local squalls are particularly unpleasant to small boats because the waves are steep, even though not particularly high. On the open ocean, very high waves can usually be ridden with little discomfort because of their relatively large wavelengths.

1.2 WAVE-FORMS

In order to simplify the theory of surface waves, we assume here that the wave-form is sinusoidal and can be represented by the curves shown in Figures 1.1 and 1.6. This assumption allows us to consider wave **displacement** (η) as simple harmonic motion, i.e. a cyclical variation in water level caused by the wave's passage. Figure 1.1 shows how the displacement varies over distance at a fixed instant in time—a 'snapshot' of the passing waves, whereas Figure 1.6 shows how wave displacement varies with time at a fixed point.

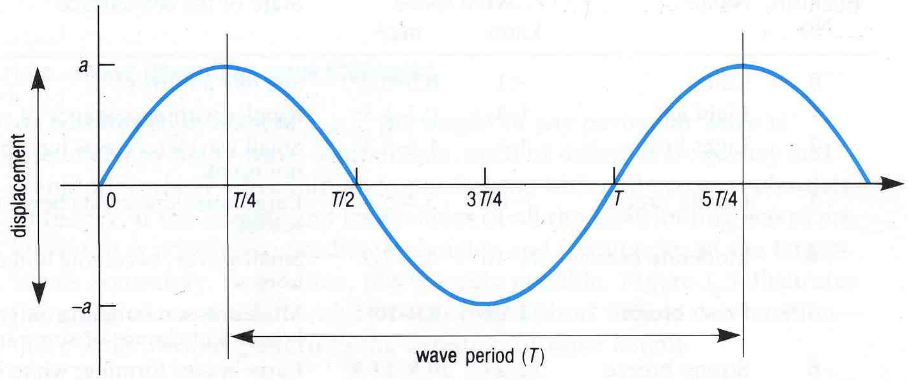


Figure 1.6 The displacement of an idealized wave at a fixed point, plotted against time.

Before examining displacement, let us remind ourselves of the relationship between period and frequency.

QUESTION 1.4 If sixteen successive wave troughs pass a fixed point during a time interval of one minute and four seconds, what is the frequency of the waves?

The displacement (η) of a wave at a fixed instant in time, or at a fixed point in space, varies between $+a$ (at the peak) and $-a$ (in the trough). Displacement is zero where $L=0$ on Figure 1.1 (and at intervals of $L/2$ from $L=0$ along the distance axis). Displacement is also zero at $T=0$ on Figure 1.6 (and at intervals of $T/2$ from $T=0$ along the time axis).

QUESTION 1.5 The peak, or crest, of a wave having a wavelength of 624 m, a frequency of $0.05\,s^{-1}$, and travelling in deep water, passes a fixed point P. What is the displacement (in terms of 'a') at P:

(a) 30 seconds after the peak has passed?

(b) 80 seconds after the peak has passed?

(c) 85 seconds after the peak has passed?

What is the displacement at a second point, Q, which is 312 m away from P in the direction of wave propagation:

(d) when the displacement at P is zero?

(e) when the displacement at P is $-a$?

(f) five seconds after a trough has passed P?

The curves shown in Figures 1.1 and 1.6 are both sinusoidal. However, most wind-generated waves do not have simple sinusoidal forms. The steeper the wave, the further it departs from a simple sine curve. Very steep waves resemble a trochoidal curve, which is illustrated in Figure 1.7.

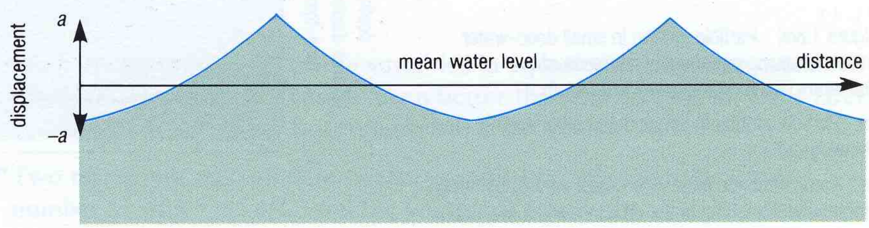


Figure 1.7 Vertical profile of two successive trochoidal waves.

A point marked on the tread of a car tyre will appear to trace out a trochoidal curve as the car is driven past an observer. Invert that pattern and you have the profile of a trochoidal water wave. We do not need to delve into the mathematical complexities of trochoidal wave-forms here, because the sinusoidal model is sufficiently accurate for an introduction to oceanic waves.

1.2.1 MOTION OF WATER PARTICLES

Water particles in a wave over deep water move in an almost closed circular path. At wave crests, the particles are moving in the same direction as wave propagation, whereas in the troughs they are moving in the opposite direction. At the surface, the orbital diameter corresponds to wave height, but the diameters decrease exponentially with increasing depth, until at a depth roughly equal to half the wavelength, the orbital diameter is negligible, and there is virtually no displacement of the water particles (Figure 1.8(a)).

It is important to realize that the orbits are only approximately circular. There is a small net component of forward motion, particularly in waves of large amplitude, so that the orbits are not quite closed, and the water, whilst in the crests, moves slightly further forward than it moves backward whilst in the troughs. This small net forward displacement of water in the direction of wave travel is termed **wave drift** (see Figure 1.8(b)). In shallow water, where depth is less than half the wavelength, the orbits become progressively flattened with depth (Figure 1.8(c)). The significance of this will be seen in Section 1.4.5, and in the Chapters on sediment movement.

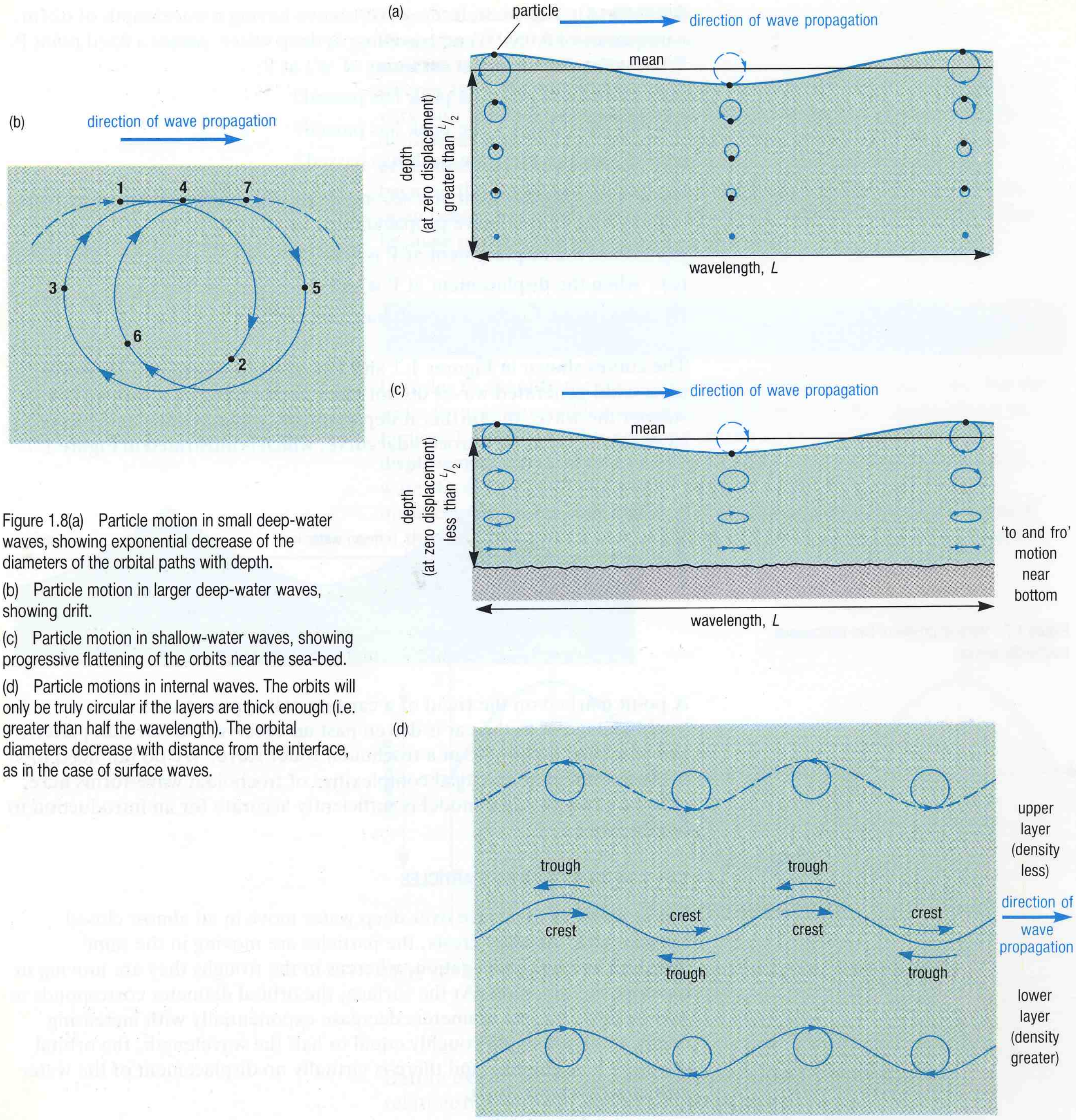


Figure 1.8(a) Particle motion in small deep-water waves, showing exponential decrease of the diameters of the orbital paths with depth.

(b) Particle motion in larger deep-water waves, showing drift.

(c) Particle motion in shallow-water waves, showing progressive flattening of the orbits near the sea-bed.

(d) Particle motions in internal waves. The orbits will only be truly circular if the layers are thick enough (i.e. greater than half the wavelength). The orbital diameters decrease with distance from the interface, as in the case of surface waves.

The nature of water particle motion in waves has some important practical applications. For example, a submarine only has to submerge about 150m to avoid the effects of even the most severe storm at sea, and knowledge of the exponential decrease of wave influence with depth has implications for the design of stable floating oil rigs.

The orbital motions relevant to internal waves are shown in Figure 1.8(d). Either side of the interface it can be seen that the water particles are moving in opposite directions, and under certain conditions may influence the movement of ships with a draught comparable with the depth to the interface.

1.2.2 SURFACE WAVE THEORY

As we have already hinted, there are mathematical relationships linking the characteristics of wavelength (L), wave period (T) and wave height (H) to wave speed in deep water and to wave energy.

First, let us consider **wave speed** (c) (the c stands for 'celerity of propagation').

Can you devise a simple formula for wave speed based on the symbols given above? Write it down before reading on.

The speed of a wave can be ascertained from the time taken for one wavelength to pass a fixed point,

$$\text{i.e.}\quad c = L/T \tag{1.1}$$

So, if we know any two of the variables in equation 1.1, we can calculate the third. However, we can do even better than that, as we shall see later, because there are other interrelationships between c, L and T.

Two terms you may meet in oceanographic literature are the **wave number** k, which is $2\pi/L$, and the **angular frequency** σ, which is $2\pi/T$.

QUESTION 1.6 How could c be expressed in terms of k and σ?

Note that the units of k are the number of waves per metre, and the units of σ are the number of cycles (waves) per second. From this information, the answer to Question 1.6 can be checked by going back to basic units, i.e. angular frequency/wave number

$$= \text{number of waves s}^{-1}/\text{number of waves m}^{-1}$$
$$= \text{wave speed (in m s}^{-1}\text{), i.e.}$$
$$c = \sigma/k \tag{1.2}$$

1.2.3 WAVE SPEED IN DEEP AND IN SHALLOW WATER

You may have noticed that, when wave speeds have been mentioned, we have been careful to state that the waves described were travelling over deep water. Thus you might have suspected that in shallow water, water depth has an effect on wave speed. If so, you were quite right. Wave speed can be represented by the equation:

$$c = \sqrt{\frac{gL}{2\pi}\tanh\left(\frac{2\pi d}{L}\right)} \tag{1.3}$$

where the acceleration due to gravity $g = 9.8\,\text{m s}^{-2}$, L = wavelength (m), and d = water depth (m). Tanh is a mathematical function known as the hyperbolic tangent. All you need to know about it in this context is that if x is small, say less than 0.05, then $\tanh x \approx x$. If x is larger than π, then $\tanh x \approx 1$.

QUESTION 1.7 Armed with equation 1.3, and the information about the tanh function, work out the answers to the following questions:

(a) What does equation 1.3 become if the water depth exceeds half the wavelength?

(b) What does equation 1.3 become if the water depth is very much smaller than L?

Let us consider the implications of your answers to Question 1.7 in terms of factors affecting wave speed:

1 In water deeper than half the wavelength, the wavelength is the only variable which affects wave speed, and equation 1.3 approximates to:

$$c = \sqrt{\frac{gL}{2\pi}} \tag{1.4}$$

2 In water very much shallower than the wavelength (in practice when $d < L/20$), water depth is the only variable which affects wave speed, and equation 1.3 approximates to:

$$c = \sqrt{gd} \tag{1.5}$$

3 When d lies between $L/20$ and $L/2$, the full form of equation 1.3 is required. Hence, to calculate wave speed you would need to know wavelength and depth, and have access to a set of hyperbolic tangent tables, or a pocket calculator with hyperbolic functions on its keyboard.

In Section 1.2.2, mention was made of interrelationships between c, T and L. The answer to Question 1.7(a) (i.e. equation 1.4) allows us to explore these relationships. We saw, in equation 1.1, that $c = L/T$, so it is possible to combine equations 1.1 and 1.4.

QUESTION 1.8 Derive an equation for wavelength (L) in terms of period (T), using equations 1.1 and 1.4.

The answer to Question 1.8 provides an equation expressing L in terms of T, i.e.

$$L = \frac{gT^2}{2\pi} \tag{1.6}$$

A similar exercise, substituting the expression obtained for L from equation 1.6 into equation 1.1, will give c in terms of T. You may like to try this for yourself—you will need it for Question 1.10(a). Thus, it is possible, given only one of the wave characteristics c, T or L, to calculate either of the other two. Moreover, by working out the numerical values of the constants involved, the equations can be simplified.

QUESTION 1.9 Simplify equation 1.6 to give a numerical relationship between L and T^2.

QUESTION 1.10(a) The period of a wave is 20 seconds. At what speed will it travel over deep water?

(b) At what speed will a wave of wavelength 312m travel over deep water?

(c) At what speeds will each of the waves referred to in (a) and (b) above travel in water of 12m depth?

The answer to Question 1.10(c) highlights an important conclusion about wave speed in shallow water. In water of a given depth, provided that depth is less than 1/20 of their wavelengths, all waves will travel at the same speed.

1.2.4 ASSUMPTIONS MADE IN SURFACE WAVE THEORY

The simple wave theory introduced in Sections 1.2.2 and 1.2.3 is a first-order approximation, and makes the following assumptions:

1 The wave shapes are sinusoidal.

2 The wave amplitudes are very small when compared with wavelengths and depths.

3 Viscosity and surface tension can be ignored.

4 Coriolis force and vorticity, both of which depend upon the Earth's rotation, can be ignored.

5 The depth is uniform, and the bottom has no bumps or hummocks.

6 The waves are not constrained or deflected by land masses, or by any other obstruction.

7 That three-dimensional waves behave in a way that is analogous to a two-dimensional model.

None of the above assumptions is valid in the strictest sense, but results predicted by using the simple model of surface wave behaviour are a close approximation to how wind-generated waves behave in practice.

1.3 WAVE DISPERSION AND GROUP SPEED

Those deep-water waves that have the greatest wavelengths and longest periods travel fastest, and thus are first to arrive in regions distant from the storm which generated them. This separation of waves by virtue of their differing rates of travel is known as **dispersion**, and equation 1.4 is sometimes known as the 'dispersion equation'.

The simple experiment of tossing a stone into a still pond shows that a band of ripples is created, which gets wider with increasing distance from the original disturbance. Ripples of greater wavelength progressively out-distance shorter ones—an example of dispersion in action. There is a second feature of the ripple band, which is not obvious at first sight. Each individual ripple travels faster than the band of ripples. A ripple appears at the back of the band, travels through it, and disappears out of the front. The speed of the band, called the **group speed**, is about half the wave speed of the individual ripples which travel through that band.

To understand the relationship between wave speed and group speed, the additive effect of two sets of waves (or wave trains) needs to be examined. If the difference between the wavelengths of two sets of waves is relatively small, the two sets will 'interfere' and produce a single set of resultant waves.

Figure 1.9 shows a simplified and idealized example of interference. Where the crests of the two wave trains coincide, the wave amplitudes are added, and the resultant wave has about twice the amplitude of the two original waves. Where the two wave trains are 'out of phase', i.e. where

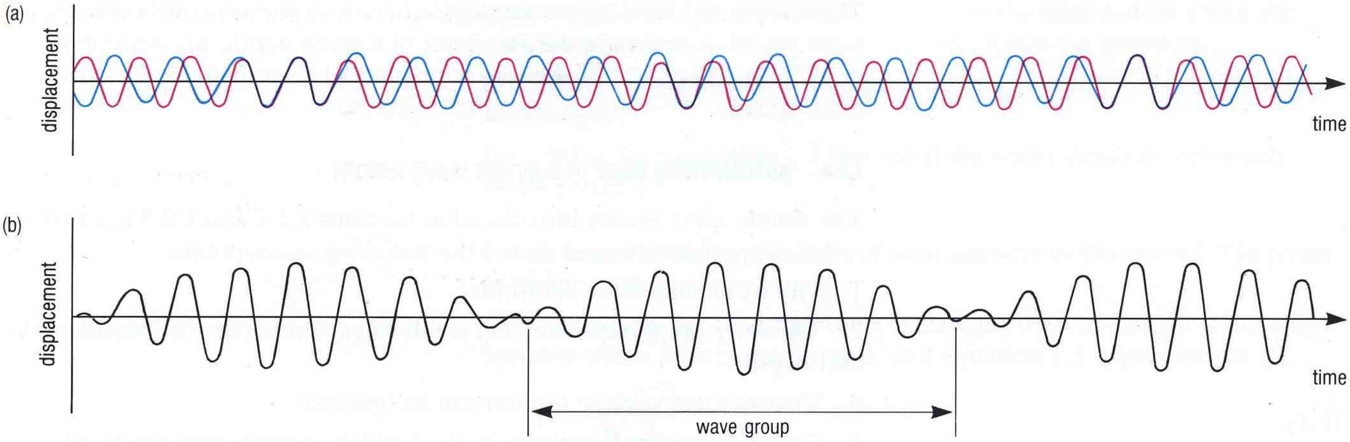


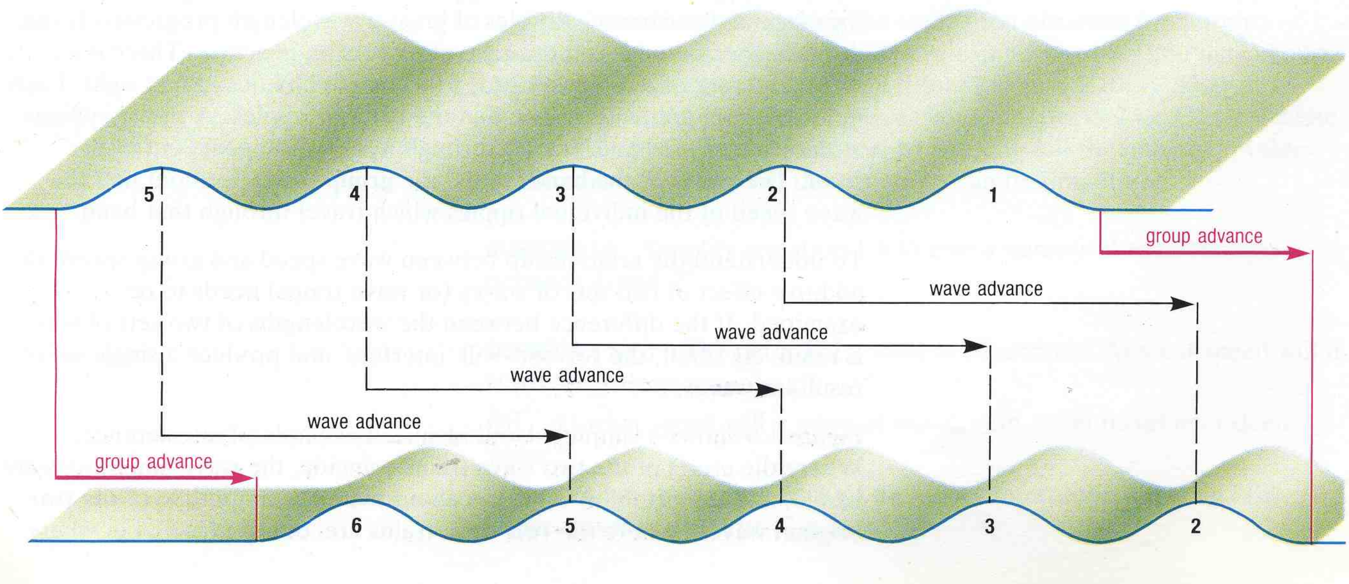
Figure 1.9(a) The merging of two wave trains (shown in red and blue) of slightly different wavelengths (but the same amplitudes), to form wave groups (b).

the crests of one wave train coincide with the troughs of the other, the amplitudes cancel out, and the water surface has minimal displacement.

The two component wave trains thus interact, each losing its individual identity, and combine to form a series of wave groups, separated by regions almost free from waves. The wave group advances more slowly than individual waves in the group, and thus in terms of the occurrence and propagation of waves, group speed is more significant than speeds of the individual waves in it. Individual waves do not persist for long in the open ocean, only as long as they take to pass through the group. Figure 1.10 shows the relationship between wave speed (sometimes called phase speed) and group speed in the open ocean.

If two sets of waves are interfering to produce a succession of wave groups, the group speed (c_g) is the difference between the two angular

Figure 1.10 The relationship between wave speed and group speed. As the wave advances from left to right, each wave moves through the group to die out at the front (e.g. wave 1), as new waves form at the rear (e.g. wave 6). In this process, the distance travelled by each individual wave as it travels from rear to front of the group is twice that travelled by the group as a whole. Hence, the wave speed is twice that of the group speed. Wave energy is contained within each group, and advances at the group speed.



frequencies (σ_1 and σ_2) divided by the difference between the two wave numbers (k_1 and k_2 respectively), i.e.

$$c_g = \frac{\sigma_1 - \sigma_2}{k_1 - k_2} \tag{1.7}$$

In Section 1.2.3, it was shown that both T and L (and hence both σ and k) can be expressed in terms of c, the wave speed. If this is done for equation 1.7, c_g can be expressed in terms of the respective speeds, c_1 and c_2, of the two wave trains. The equation obtained is:

$$c_g = \frac{c_1 \times c_2}{c_1 + c_2} \tag{1.8}$$

If c_1 is nearly equal to c_2, then equation 1.8 simplifies to:

$$c_g \approx c^2/2c$$

$$\text{or} \quad c_g \approx c/2 \tag{1.9}$$

where c is the average speed of the two wave trains.

What happens to group speed when waves enter shallow water?

Equation 1.3 shows that as the water becomes shallower, wavelength becomes less important, and depth more important, in determining wave speed. As a result, wave speed in shoaling water becomes closer to group speed. Eventually, at depths less than $L/20$, all waves travel at the same depth-determined speed, there will be no wave–wave interference, and therefore in effect each wave will represent its own 'group'. Thus, in shallow water, group speed can be regarded as equal to wave speed.

1.4 WAVE ENERGY

The energy possessed by a wave is in two forms:

1 kinetic energy, which is the energy inherent in the orbital motion of the water particles; and

2 potential energy possessed by the particles when they are displaced from their mean position.

The total energy (E) per unit area of a wave is given by:

$$E = 1/8\ (\rho g H^2) \tag{1.10}$$

where ρ is the density of the water (in kg m^{-3}), g is $9.8\,\text{m s}^{-2}$, and H is the wave height (m). The energy (E) is then in joules per square metre (J m^{-2}).

QUESTION 1.11 Would the total energy of a wave be doubled if its amplitude were doubled?

Equation 1.10 shows that wave energy is proportional to the square of the wave height.

1.4.1 PROPAGATION OF WAVE ENERGY

Figures 1.9 and 1.10 show that, in deep water, waves travel in groups, with areas of minimal disturbance between groups. Individual waves die out at the front of each group. It is obvious that no energy is being transmitted across regions where there are no waves, i.e. in between the groups. Energy transmission is maximal where the waves in the group reach maximum size. It follows that the energy is contained within the wave group, and is propagated at the group speed. The rate at which energy is propagated per unit length of wave crest is called **wave power**, and is the product of group speed (c_g), and wave energy per unit area (E).

QUESTION 1.12(a) In the case of waves over deep water, what is the energy per square metre of a wave field made up of waves with an average amplitude of 1.3 m? (Use $\rho = 1.03 \times 10^3 \text{kg m}^{-3}$.)

(b) What would be the wave power in kW per metre of crest length if the waves had a steepness of 0.04? (1 watt $= 1 \text{J s}^{-1}$, and one kilowatt (kW) $= 10^3$ W.)

Wave power has been seen by some as a possible source of pollution-free 'alternative energy', and has been used for some time on a small scale to recharge batteries on buoys carrying navigation lights. Harnessing wave energy on a large scale presents a number of problems.

1 Prevailing sea conditions must ensure a supply of waves with amplitudes sufficient to make conversion worthwhile.

2 Installations must not be a hazard to navigation, or to marine ecosystems. The nature of wave energy is such that rows of converters many kilometres in length are needed to generate amounts of electricity comparable with conventional power stations. These would form offshore barrages which might interfere with shipping routes, although sea conditions would be made calmer on the shoreward side. Calmer conditions, however, lead to reduced water circulation, less sediment transport, and increased growth of quiet-water plants and animals. Pollutants are less easily flushed away from such an environment.

3 The capital cost of such floating power stations and their related energy transmission and storage systems is enormous. Installations need to be robust enough to withstand storm conditions, yet sensitive enough to be able to generate power from a wide range of wave sizes. Such conditions are expensive to meet, and make it difficult for large-scale wave-energy schemes to be as cost effective as conventional energy sources. Relatively small-scale utilization of wave power is more feasible, as has been demonstrated by the Norwegians, who in 1985 brought into operation a wave-powered generator of 850 kW. This machine was sited on the west coast of Norway, where waves funnelled into a narrow bay and increased substantially in height and thus in energy.

1.4.2 SWELL

The sea-surface is rarely still. Even when there is no wind, and the sea 'looks like a mirror', a careful observer will notice waves of very large wavelength (say 300 to 600 m) and only a few centimetres amplitude. At other times, a sea may include locally generated waves of small wavelength, and travelling through these waves, possibly at a large angle to the wind, other waves of much greater wavelength. Such long waves are known as **swell**, which is simply defined as waves that have been generated elsewhere and have travelled far from their place of origin. Systematic observations show that local winds and waves have very little effect on the size and progress of swell waves. Swell seems able to pass through locally generated seas without hindrance or interaction. This is because once swell waves have left the storm area, their wave height gradually diminishes, due to attenuation (Section 1.4.3). Once wave height has diminished to a few centimetres, swell waves are not steep enough to be significantly influenced by the wind.

In the ocean we find waves travelling in many directions, resulting in an apparently confused sea. To achieve a complete description of such a sea-surface, the amplitude, frequency, and direction of travel of each component would be needed. The energy distribution of the sea-surface could then be calculated, but, as you might imagine, such a complex process would require expensive equipment to measure the wave characteristics, and computer facilities to perform the necessary calculations.

One or more components of a confused sea may be long waves or swell resulting from distant storms. In practice, about 90% of the energy of the sea-surface propagates within an angle of 45° either side of the wind direction. Consequently, waves generated by a storm in a localized region of a large ocean radiate outwards as a segment of a circle (Figure 1.11). As the circumference of the circle increases, so energy per unit length of wave must decrease, so that the total energy of the segment remains the same.

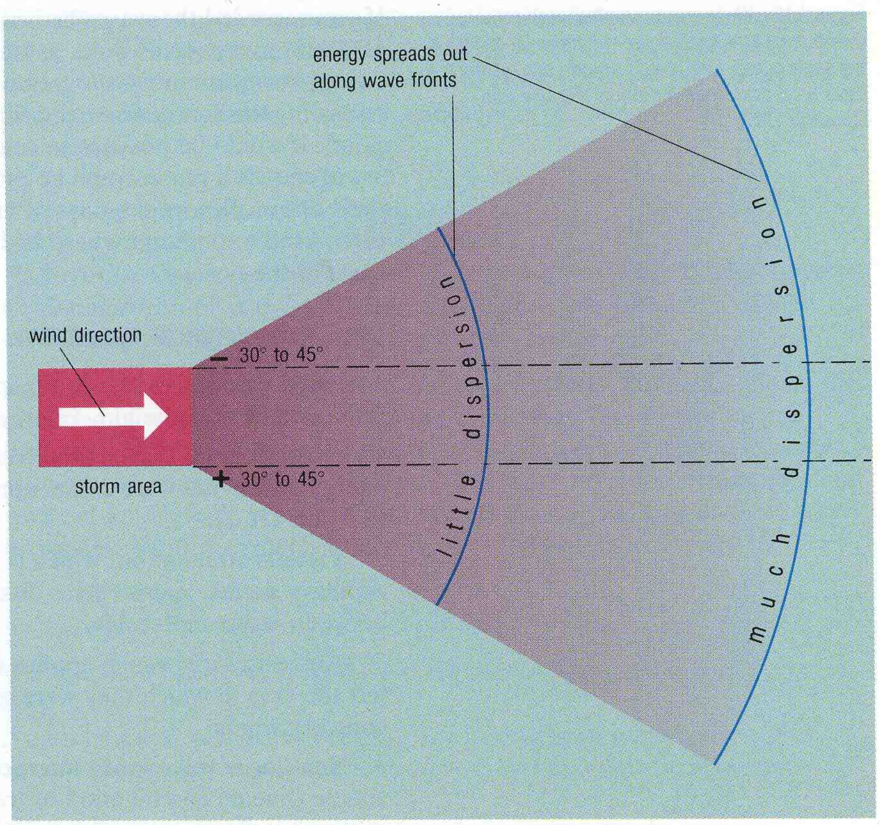


Figure 1.11 The spreading of swell from a storm centre, showing the area in which swell might be expected. As distance from the storm increases, the length of the wave crest increases, with a corresponding decrease in energy per unit length of wave. (By convention, direction is measured clockwise from North, hence the position of the plus and minus signs.)

The waves with the longest periods travel fastest, and progressively out-distance waves of higher frequencies (shorter periods). Near to the storm, dispersion is unlikely to be well defined, but the further one moves from the storm location, the more clearly separated waves of differing frequencies become.

QUESTION 1.13 Figure 1.12 shows two wave-energy spectra, (a) and (b) (*cf.* Figure 1.4). One represents the energy of the wave field in a storm-generating area; and the other represents the energy of the wave field in an area far away from the storm, but receiving swell from it. Which of the two profiles represents which situation?

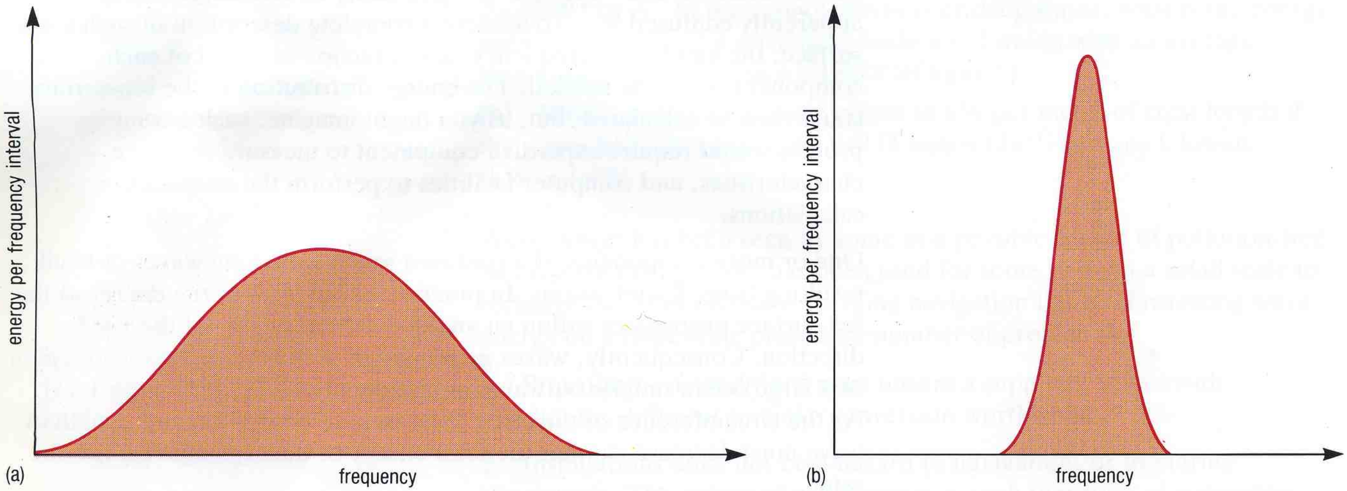


Figure 1.12 Wave energy spectra, each collected over a short time interval, for two areas (a) and (b) in the same ocean. One area is a storm centre, and the other is far away from the storm. (For use with Question 1.13.)

If you recorded the waves arriving from a storm a great distance (over 1000km) away, you would, as time progressed, see the peak in the wave energy spectrum move progressively towards higher frequencies. By recording the frequencies of each of a series of swell waves arriving at a point, it would be possible to calculate each of their speeds. From the set of speeds a graph could be plotted to estimate the time and place of their origin. Before the days of meteorological satellites, this method was often used to pinpoint where and when storms had occurred in remote parts of the oceans.

1.4.3 ATTENUATION OF WAVE ENERGY

Attenuation involves loss or dissipation of wave energy, resulting in a reduction of wave height. Energy is dissipated in four main ways:

1 White-capping, which involves transfer of wave energy to the kinetic energy of moving water, thus reinforcing the wind-driven surface current (Section 1.1.2).

2 Viscous attenuation, which is only important for very high frequency capillary waves, and involves dissipation of energy into heat by friction between water molecules.

3 Air resistance, which applies to large steep waves soon after they have left the area in which they were generated and enter regions of calm or contrary winds.

4 **Non-linear wave–wave interaction**, which is more complicated than the simple (linear) combination of frequencies to produce wave groups as outlined in Section 1.3.

Non-linear interaction appears to be most important in the frequency range of 0.2 to $0.3\,s^{-1}$. Groups of three or four frequencies can interact in complex non-linear ways, to transfer energy to waves of both higher and lower frequencies. A rough but useful analogy is that of the collision of two drops of water. A linear combination would simply involve the two drops coalescing into one big drop, whereas a non-linear combination is akin to a collision between the drops so that they split into a number of drops of differing sizes. The total amount of water in the drops (analogous to the total amount of energy in the waves) is the same before and after the collision.

Thus non-linear wave–wave interaction involves no loss of energy in itself, because energy is simply 'swapped' between different frequencies. However, the total amount of energy available for such 'swapping' will gradually decrease, because higher frequency waves are more likely to dissipate energy in the methods described under 1 and 2 above. For example, higher frequency waves are likely to be steep, and thus more prone to white-capping. As we saw in Section 1.4.2, in the case of established swell waves there is very little loss of wave energy apart from that caused by spreading over a progressively wider front (Figure 1.11). Wave attenuation is greatest in the storm-generating area, where there are waves of many frequencies, and hence more opportunities for energy exchange between waves of frequencies in the range 0.2 to $0.3\,s^{-1}$.

1.4.4 WAVE REFRACTION

Figure 1.13 shows an idealized linear wave crest (length s_1, between A and B) approaching a shoreline at an angle. Because the waves are travelling over shallow water, their speed is depth-determined (equation 1.5, $c=\sqrt{gd}$). Depth at A exceeds depth at B, hence the wave at A will travel faster than the wave at B. This will tend to 'swing' the wave crest to an alignment parallel to the depth contours; a phenomenon known as **refraction**.

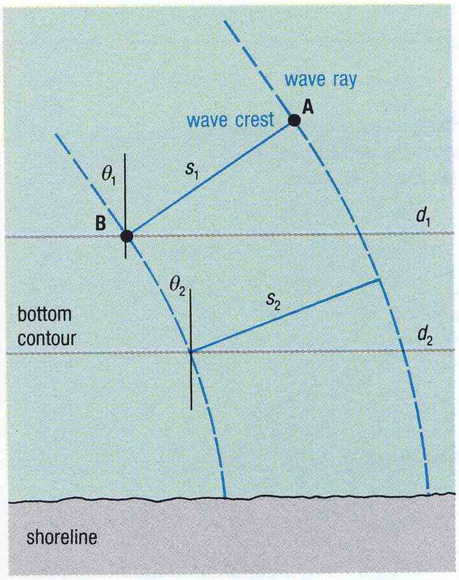


Figure 1.13 Plan view illustrating the relationship between wave approach angle (θ), water depth (d), and wave crest length (s). The wave rays (broken blue lines), are normal to the wave crests, and are the paths followed by points on the wave crests. For further explanation, see text.

Can the extent of refraction be quantified?

Refraction of waves in progressively shallowing water can be described by a relationship similar to Snell's law, which describes refraction of light rays through materials of different refractive indices.

Rays can be drawn perpendicular to the wave crests, and will indicate the direction of wave movement. The angles between these wave rays and lines drawn perpendicular to the depth contours can be related to wave speeds at various depths. In Figure 1.13, a wave ray approaching shoaling water at an angle θ_1, where water depth is d_1, will be at an angle θ_2 when it reaches depth d_2. Angles θ_1 and θ_2 are related to wave speed by:

$$\frac{\sin\theta_1}{\sin\theta_2}=\frac{c_1}{c_2}=\frac{\sqrt{gd_1}}{\sqrt{gd_2}}=\frac{\sqrt{d_1}}{\sqrt{d_2}}=\sqrt{\frac{d_1}{d_2}} \qquad (1.11)$$

where c_1 and c_2 are the respective wave speeds at depths d_1 and d_2.

You might ask: why go to the trouble of drawing perpendiculars? Why not simply use the angles between wave crests and bottom contours?

Well, of course, one could do that, and obtain exactly the same relationships between the relevant angles, depths and wave speeds. However, wave rays are often more useful than wave crests in determining regions that are likely to experience high waves due to the effects of refraction.

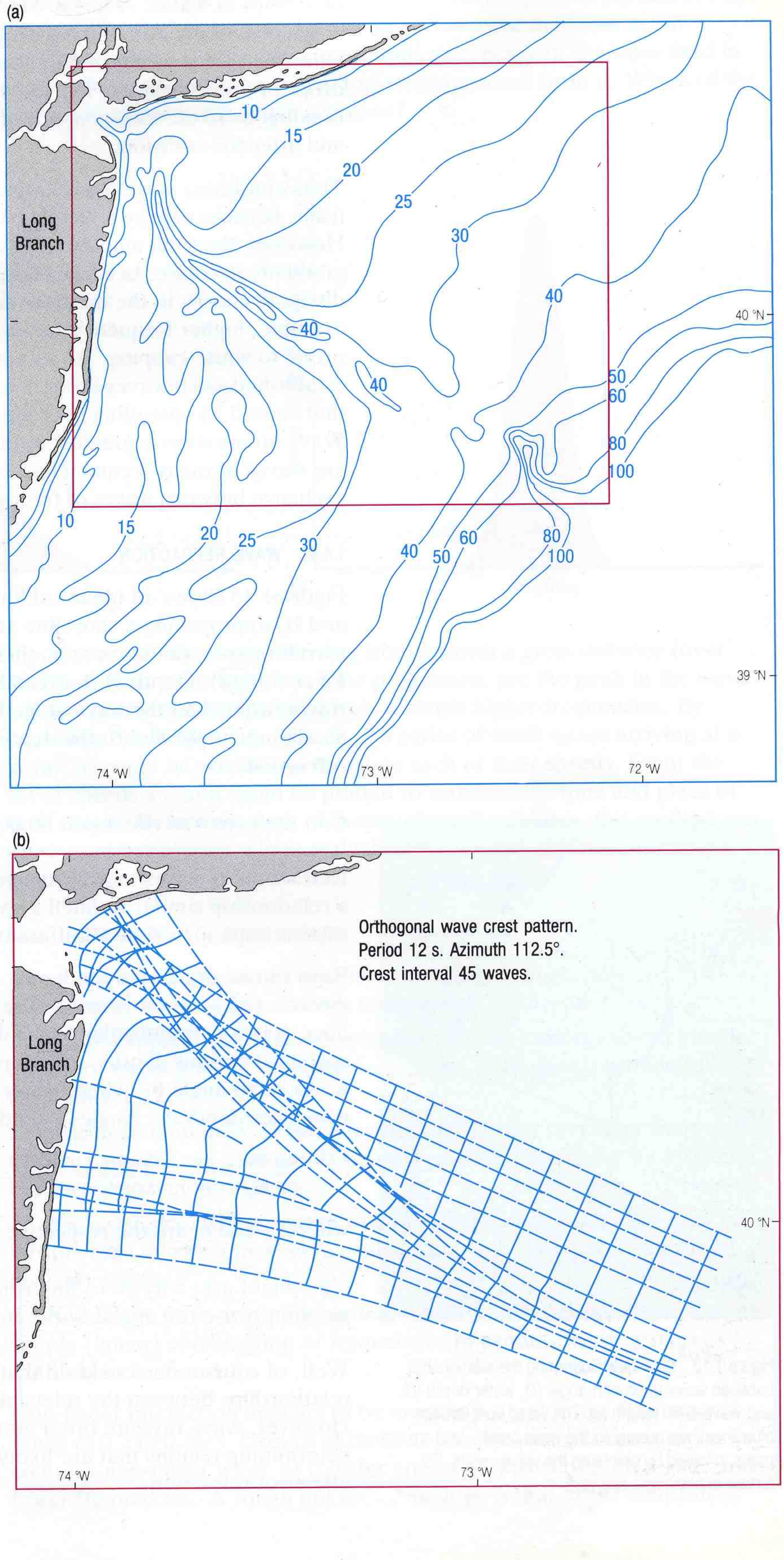


Figure 1.14(a) Bathymetric map of the continental shelf off New York harbour, at the mouth of the Hudson River, on the Atlantic coast of the USA. The rectangle shows the area of map (b). The position of the Hudson Canyon can be deduced from the submarine contours (in fathoms). (b) Orthogonal wave crest (wave front) pattern of part of Long Branch beach. The crest intervals are of 45 waves of period 12s.

Consider a length, s_1, of ideal wave crest, with energy per unit length E_1, which is bounded by two wave rays, as in Figure 1.13. To a first approximation, we may assume that the total energy of the wave crest between these two rays would remain constant as the wave progresses. Therefore, if the two rays converge, the same amount of energy is contained within a shorter length of wave crest, so that, for the total wave energy to remain constant, the wave would have to become higher (equation 1.10). If the wave rays were to diverge then the wave would become lower.

If the two wave rays, as they finally approach the shore, are separated by a length s_2, as in Figure 1.13, and if the wave energy is conserved, then the final wave energy must equal the initial wave energy, i.e. $E_1s_1 = E_2s_2$, or in terms of wave heights:

$$H_1^2s_1 = H_2^2s_2 \tag{1.12}$$

(Remember $E = 1/8\ (\rho gH^2)$, eqn. 1.10.)

Note that for simplicity, s_2 in Figure 1.13 is the same length as s_1. However, it is common for wave rays to converge or diverge. Wave refraction diagrams can be plotted for a region by using the wave of most common period and the most common direction of approach, and areas in which wave rays are focused or defocused can be identified.

QUESTION 1.14 Figure 1.14 is a bathymetric map and storm wave refraction diagram for the Hudson River submarine canyon on the Atlantic coast of the USA. In what area covered by the refraction diagram would you advise fishermen to leave their boats with least likelihood of major damage, and why?

We can estimate increase or decrease in wave size by measuring the distances between wave rays, and applying equation 1.12. This method is quite useful provided wave rays neither approach each other too closely, nor cross over, as in these cases the waves become high, steep and unstable, and so simple wave theory becomes inadequate.

1.4.5 WAVES APPROACHING THE SHORE

As you have seen in the previous Section, refraction can change wave height, but it is also apparent that waves coming straight onto a beach increase in height and steepness until they break.

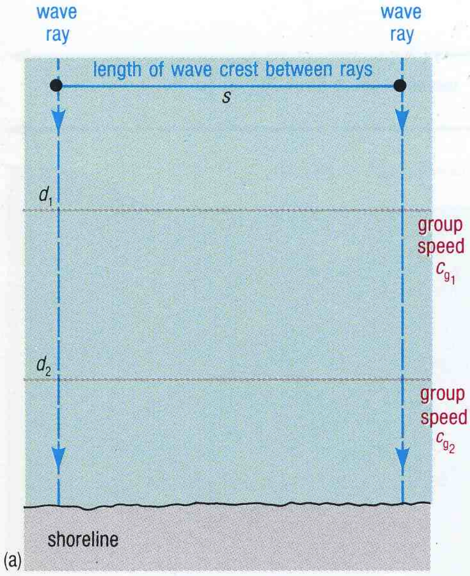


Figure 1.15(a) Plan view illustrating changes in energy as waves approach the shore. For explanation, see text.

Figure 1.15(a) shows a length of wave crest, s, which is directly approaching a beach. As the water is shoaling, the wave crest passes a first point d_1, where the water is deeper than at a second point d_2, nearer the shore. We assume that the amount of energy within this length of wave crest remains constant, the wave is not yet ready to break, and that water depth is less than 1/20 of the wavelength (i.e. equation 1.5 applies: $c = \sqrt{gd}$). Because wave speed in shallow water is related to depth, the speed c_1 at depth d_1 is greater than the speed c_2 at depth d_2. If energy remains constant per unit length of wave crest, then

$$E_1c_1s = E_2c_2s$$

or

$$\frac{E_2}{E_1} = \frac{c_1}{c_2} \tag{1.13}$$

and because energy is proportional to the square of the wave height (equation 1.10) then we can write

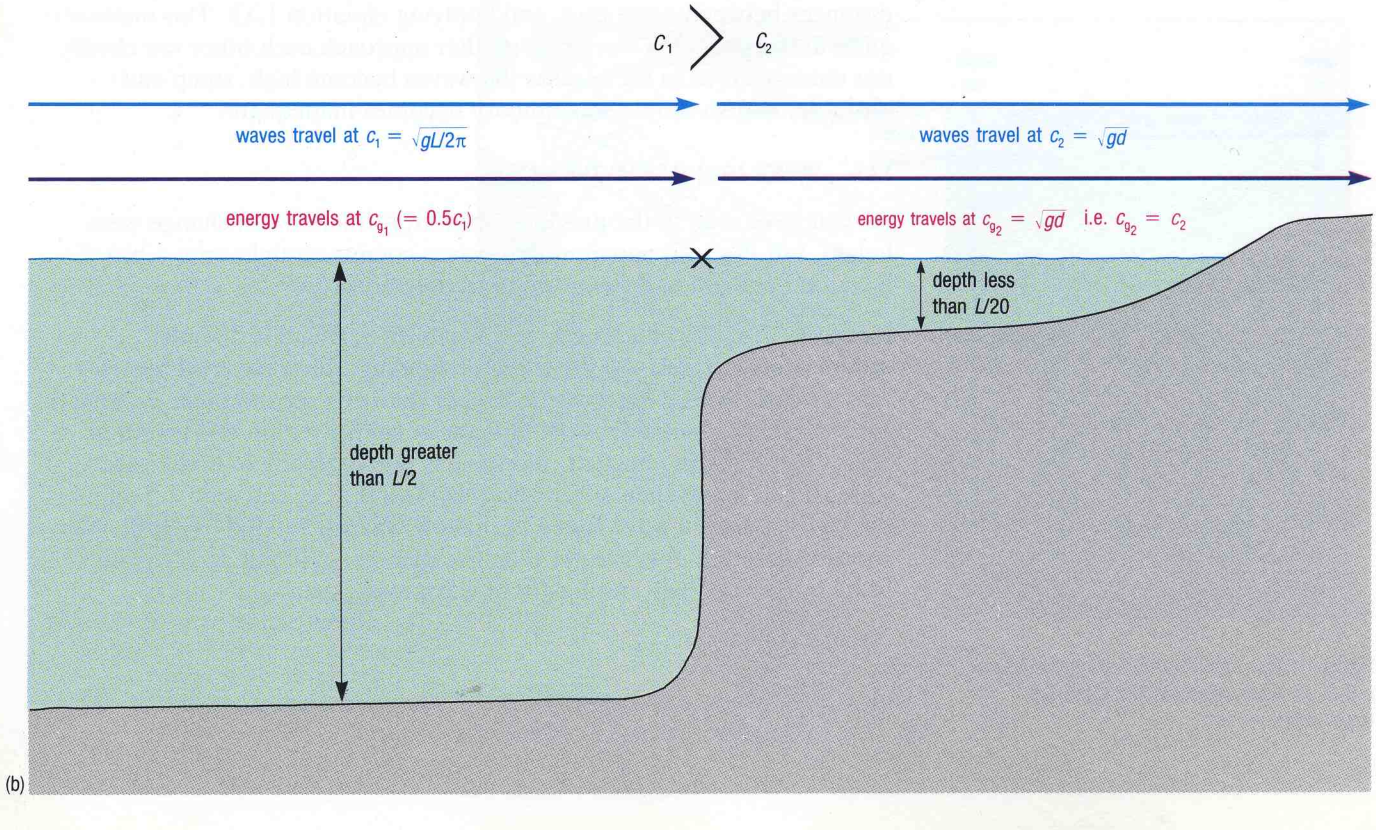
$$\frac{E_2}{E_1} = \frac{c_1}{c_2} = \frac{H_2^2}{H_1^2} \tag{1.14}$$

Thus, both wave energy and the square of the wave height are inversely proportional to wave speed in shallow water.

This relationship is straightforward once the wave has entered shallow water. But what happens to energy in waves travelling at group speed over deep water as they move into shallow water where speed is wholly depth-determined?

This is quite a difficult question, best answered by considering the highly simplified case illustrated in Figure 1.15(b). Imagine waves travelling shoreward over deep water (depth greater than half the wavelength). Wave speed is then governed solely by wavelength (equation 1.4, $c = \sqrt{gL/2\pi}$). The energy is being propagated at the group speed (c_g) which is approximately half the wave speed (c). Once the waves have moved into shallow water, wave speed becomes governed solely by depth and is much reduced. Remember from Section 1.3 that group speed is equal to wave speed in shallow water. The rate at which energy arrives from offshore of point X on Figure 1.15(b) is equal to the rate at which

Figure 1.15(b) Vertical profile of a highly simplified shoreline. The energy is being brought in from offshore at the same rate as it is being removed as the waves break. It follows that if c_{g1} is greater than c_{g2}, then there must be more energy per unit length of wave, and a greater wave height, in those waves travelling at c_{g2}.



energy moves inshore of X, so if the group speed in shallow water is less than half the original wave speed (and hence less than the original group speed) in deep water, the waves will show corresponding increases in height and in energy. Of course, a 'real' coastline shows a much less abrupt change from deep to shallow water than does Figure 1.15(b), and calculations of intermediate values of group speeds, wave speeds, heights and energies become more complicated, because at depths between 1/2 and 1/20 of the wavelength, equation 1.3 must be used.

As waves move into shallow water, the circular orbits of the water particles become flattened (Figure 1.8(c)) and some wave energy will be used in moving sediments to and fro on the sea-bed. The shallower the slope of the immediate offshore region, the more energy will be lost from waves before they finally break.

1.4.6 WAVES BREAKING UPON THE SHORE

A breaking wave is a highly complex system. Even some distance before the wave breaks, its shape is substantially distorted from a simple sinusoidal wave. Hence the mathematical model of such a wave is more complicated than that we have assumed in this Chapter. As a wave breaks, the energy it received from the wind is dissipated. Some energy is reflected back out to sea, the amount depending upon the slope of the beach—the shallower the angle of the beach slope, the less energy is reflected. Most of the energy is dissipated as heat in the final small-scale mixing of foaming water, sand and shingle. Some energy is used in fracturing large rock or mineral particles into smaller ones, and yet more may be used to increase the height and hence the potential energy of the beach form. This last aspect depends upon the type of waves. Small gentle waves and swell tend to build up beaches, whereas storm waves tear them down.

Four major types of breaker are seen:

1 Spilling breakers are characterized by foam and turbulence at the wave crest. Spilling usually starts some distance from shore and is caused when a layer of water at the crest moves forward faster than the wave as a whole. Foam eventually covers the leading face of the wave. Such waves are characteristic of a gently sloping shoreline. A tidal bore (Section 2.4.3) is an extreme form of a spilling breaker. Breakers seen on beaches *during* a storm, when the waves are steep and short, are of the spilling type. They dissipate their energy gradually as the top of the wave spills down the front of the crest, which gives a violent and formidable aspect to the sea because of the more extended period of breaking.

2 Plunging breakers are the most spectacular type. The classical form, much beloved by surf-riders, is arched, with a convex back and a concave front. The crest curls over and plunges downwards with considerable force, dissipating its energy over a short distance. Plunging breakers on beaches of relatively gentle slope are usually associated with the long swells generated by distant storms. Locally generated storm waves seldom develop into plunging breakers on gently sloping beaches, but may do so on steeper ones.

3 Collapsing breakers are similar to plunging breakers, except that instead of the crest curling over, the front face collapses. Such breakers occur on beaches with moderately steep slopes, and under moderate wind conditions.

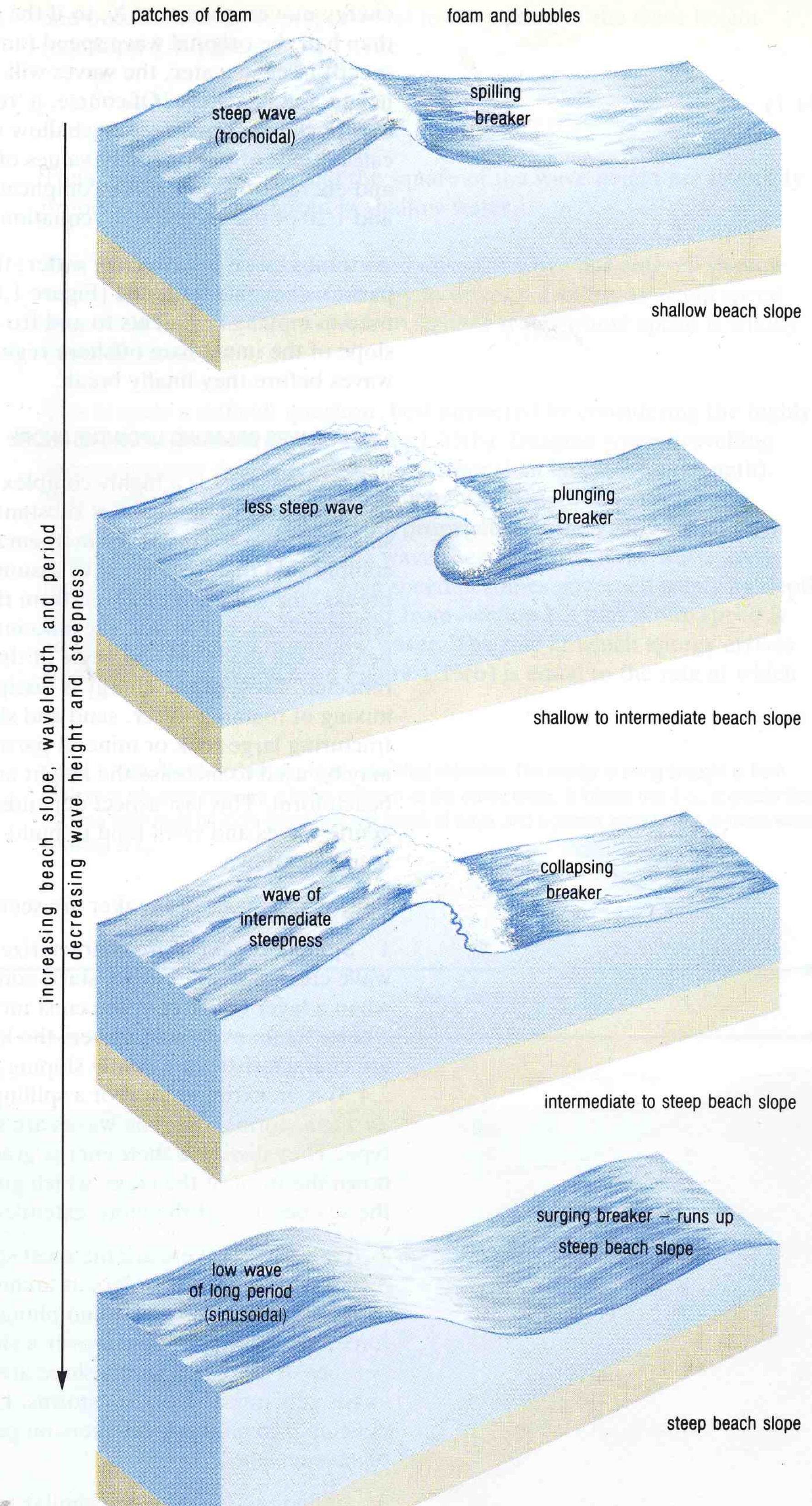


Figure 1.16 Four types of breaker and their relationships to beach slope, wave period, length, height and steepness.

4 Surging breakers are found on the very steepest beaches. Surging breakers are typically formed from long, low waves, and the front faces and crests remain relatively unbroken as the waves slide up the beach.

Figure 1.16 illustrates the relationship between wave steepness, beach steepness and breaker type.

QUESTION 1.15 If you observed plunging breakers on a beach and walked along towards a region where the beach became steeper, what different types of breaker might you expect to see?

From the description, Figure 1.16 and the answer to Question 1.15, it can be seen that the four types of breaker form a continuous series. The spilling breaker, characteristic of shallow beaches and steep waves (i.e. with short periods and large amplitudes), forms one end member. At the other end of the series is the surging breaker, characteristic of steep beaches and of waves with long periods and small amplitudes.

For a given beach, the arrival of waves steeper than usual will tend to give a type of breaker nearer the 'spilling' end of the series, whereas calmer weather favours the surging type.

1.5 WAVES OF UNUSUAL CHARACTER

Waves of unusual character may result from any one of a number of conditions, such as a particular combination of wave frequencies; the constraining effect of adjacent land masses; interaction between waves and ocean currents; or a submarine earthquake. The destructive effects of abnormally large waves are well known, and hence prediction of where and when they will occur is of extreme importance to all who live or work beside or upon the sea.

1.5.1 WAVES AND CURRENTS

Anyone who regularly sails a small boat into and out of estuaries will be well aware that at certain states of the tide the waves can become abnormally large and uncomfortable. Such large waves are usually associated with waves propagating against an ebbing tide. Because the strength of the tidal current varies with position as well as with time, waves propagating into an estuary during an ebb tide often advance into progressively stronger counter-currents.

Consider a simple system of deep-water waves, moving from a region with little or no current (A) into another region (B) where there is a current flowing parallel to the direction of wave propagation. Imagine two points, one in region A and one in region B, each of which are fixed with respect to the sea-bed. The number of waves passing each point in a given time must be the same, otherwise waves would either have to disappear, or be generated, between the two points. In other words, the wave period must be the same at each point.

How will wavelength and wave height be affected if the current is flowing (a) with or (b) against the direction of wave propagation?

Clearly, a current flowing with the waves will have the effect of increasing the speed of the waves, although the wave period (T) must remain constant, i.e.

$$T = \frac{L_0}{c_0} = \frac{L}{c+u} \tag{1.15}$$

where L_0 = wavelength when current is zero;
c_0 = wave speed when current is zero;
L = wavelength in the current;
c = wave speed in the current;
u = speed of the current.

Because $c+u$ is greater than c_0, then for T to remain constant, L must be greater than L_0, i.e. the waves get longer. Moreover, the waves get correspondingly lower, because the rate of energy transfer depends upon group speed (half wave speed) and wave height. If the rate of energy transfer is to remain constant, then as speed increases, wave height must decrease. However, in practice, not all of the wave energy is retained in the wave system: some is transferred to the current, causing wave height to decrease still further.

Conversely, if the current flows counter to the direction of wave propagation, then L will decrease and the waves will get shorter and higher. Wave height will be further increased as a result of energy gained from the current. In theory, a point could be reached where wave speed is reduced to zero, so that a giant wave builds up to an infinite height (it can be shown mathematically that this occurs when the counter-current exceeds half the group speed of the waves in still water). However, in practice, as waves propagate against a counter-current of ever-increasing strength, the waves become shorter, steeper and higher until they become unstable and break, so that waves do not propagate against a counter-current of more than half their group speed.

Consider a wide estuary or fjord with a relatively narrow inlet from the sea. The only ocean waves that disturb this estuary during an ebbing tide are those that have speeds sufficiently high to overcome the effects of the counter-current. A current can also refract waves, i.e. change the direction of propagation. In such situations, the refraction diagram of the wave rays can be plotted in a similar way to that outlined in Section 1.4.4 (see also Section 1.5.2 and Figure 1.17(b)).

1.5.2 GIANT WAVES

The cultures of all seafaring nations abound with legends of ships being swamped by gigantic waves, and of sightings of waves of unbelievable size.

QUESTION 1.16 An elderly ex-seaman, in his cups, claims to have seen gigantic waves in the Southern Ocean, successive peaks of which took 30 seconds to pass, and which had wavelengths twice as long as his ship. Can you believe him?

Before dismissing the sailor's claim in Question 1.16 as a tall story, let us examine it more closely. Let us suppose his ship was travelling in the same direction as the waves and was being *overtaken* by them, and he had made the simple mistake of not taking account of the ship's velocity with respect to the waves when timing the intervals between successive peaks.

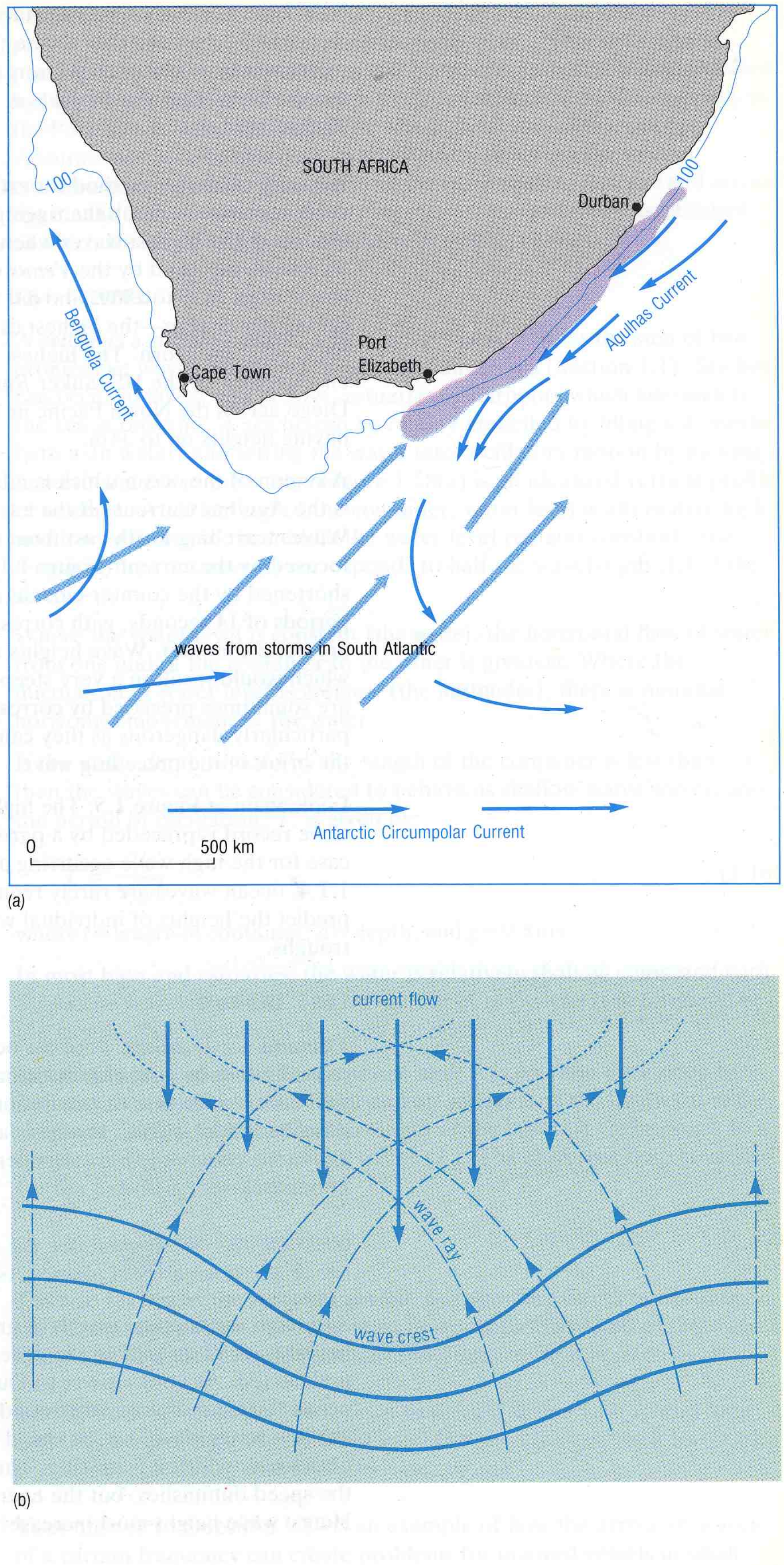


Figure 1.17(a) Giant waves have been recorded in the vicinity of the Agulhas Current (area shaded in red). Dark blue arrows show current direction, and light blue arrows indicate the direction of wave movement. The thin blue line is the 100 fathom contour.
(b) Diagram to illustrate how wave energy might be focused by lateral shear in a counter-current. The lengths of the dark blue arrows are proportional to the current velocity, curved solid lines are wave crests, and curved broken arrows are wave rays, with dashed extensions to indicate how waves could become trapped within the current if conditions were right.

QUESTION 1.17 Further conversation with the seaman established that his ship was the cruiser HMS *Exeter* (1929–42), which at the time of the incident was steaming at 23 knots (11.8 m s^{-1}) in the same direction as the waves. Given that the *Exeter* was 575 feet in length (175 m), can you believe him now?

An early objective method for estimating the height of large waves was to send a seaman to climb the rigging until he could just see the horizon over the top of the highest waves when the ship was in a wave trough. This technique was used by the *Venus* during her circumnavigation of the world from 1836 to 1839. She did not meet any particularly high waves during her voyage—the highest estimated by this method was about 8 m high, off Cape Horn. The highest reliably measured wave was that encountered by the US tanker *Ramapo*, en route from Manila to San Diego across the North Pacific in 1933. The ship was overtaken by waves having heights up to 34 m.

A region of the ocean which is infamous for encounters with giant waves is the Agulhas Current off the east coast of South Africa (Figure 1.17(a)). Waves travelling north-east from the southern Atlantic Ocean tend to be focused by the current (Figure 1.17(b)), and are further steepened and shortened by the counter-current effect outlined in Section 1.5.1. Wave periods of 14 seconds, with corresponding wavelengths of about 300 m, are quite common. Wave heights in this region can be of the order of 30 m which would result in a very steep wave (0.1). Such high and steep waves are sometimes preceded by correspondingly deep troughs, which are particularly dangerous as they can only be seen by vessels which are on the brink of the preceding wave.

Look again at Figure 1.5. The high wave occurring 122 seconds into the wave record is preceded by a particularly deep trough, but that is not the case for the high wave occurring at 173 seconds. As we saw in Section 1.1.4, ocean waves are rarely regular, and it is usually not possible to predict the heights of individual waves, nor the depths of individual troughs.

1.5.3 TSUNAMIS

Tsunami is a Japanese word for ocean waves of very great wavelength, caused either by a seismic disturbance, or by slumping of submarine sediment masses due to gravitational instability. Although commonly miscalled 'tidal waves', tsunamis are not caused by tidal influences. Tsunamis commonly have wavelengths of the order of hundreds of kilometres.

QUESTION 1.18 What would be the speed of a tsunami across the open ocean above an abyssal plain? (Assume the average depth is 5.5 km.)

Although the tsunami travels at great speed in the open ocean, its wave height is small, usually in the order of one metre, and often remains undetected. As your answer to Question 1.18 indicates, even in the open ocean the ratio of wavelength to depth is such that a tsunami travels as a shallow-water wave, i.e. its speed is always governed by the depth of ocean over which it is passing. Thus, on reaching even shallower water, the speed diminishes, but the energy in the wave remains the same. Hence wave height must increase.

Great destruction can be wreaked by a tsunami. It is not unknown for people on board ships at anchor offshore to be unaware of a tsunami passing beneath them, but to witness the adjacent shoreline being pounded by large waves only a few seconds later. Tsunamis occur most frequently in the Pacific, because that ocean experiences frequent seismic activity. Accurate earthquake detection can give warning of the approach of tsunamis to coasts some distance from the earthquakes. Around and across the Pacific Ocean, a system of warning stations has long been established, of which Honolulu is the administrative and geographical centre.

1.5.4 SEICHES

A **seiche** is a standing wave, which can be considered as the sum of two progressive waves, travelling in opposite directions (Section 1.1). Seiches can occur in lakes, and in bays, estuaries or harbours which are open to the sea at one end. A seiche can be readily modelled by filling a domestic bath with water, and setting the water into oscillatory motion by moving a hand to and fro in the water. Figure 1.18(a) is an idealized vertical profile of a seiche. At either end of the container, water level is alternately high and low, whereas in the middle the water level remains constant. The length of the container (l) corresponds to half the wavelength (L) of the seiche.

Where the water level is constant (the **node**), the horizontal flow of water from one end of the container to the other is greatest. Where the fluctuation of water level is greatest (the **antinodes**), there is minimal horizontal movement of the water.

If the water depth divided by the length of the container is less than 0.1, then the waves can be considered to behave as shallow water waves, and the period of oscillation, T, is given by:

$$T = \frac{2l}{\sqrt{gd}} \tag{1.16}$$

where l = length of container; d = depth; and $g = 9.8\,\mathrm{m\,s^{-2}}$.

In most bays and estuaries, the water is relatively shallow compared with the seiche wavelength (L), and the period of the seiche is determined by the length of the basin and the depth of water in it.

In some basins, open to the sea at one end, it is possible for a node to occur at the entrance to the basin and an antinode at the landward end (Figure 1.18(b)). In this case, the length of the basin (l) corresponds to a quarter of the wavelength of the seiche (L). The corresponding equation for the period is therefore:

$$T = \frac{4l}{\sqrt{gd}} \tag{1.17}$$

T is also known as the **resonant period**. For standing waves to develop, the resonant period of the basin must be equal to the period of the wave motion or to a small whole number of multiples of that period.

QUESTION 1.19 A small harbour, open to the sea at one end, is 90 m long and 10 m deep at high water. What would be the effect of swell waves of period 18 seconds arriving at the harbour mouth?

Your answer to Question 1.19 is an example of how the arrival of waves of a certain frequency can create problems for moored vessels in small

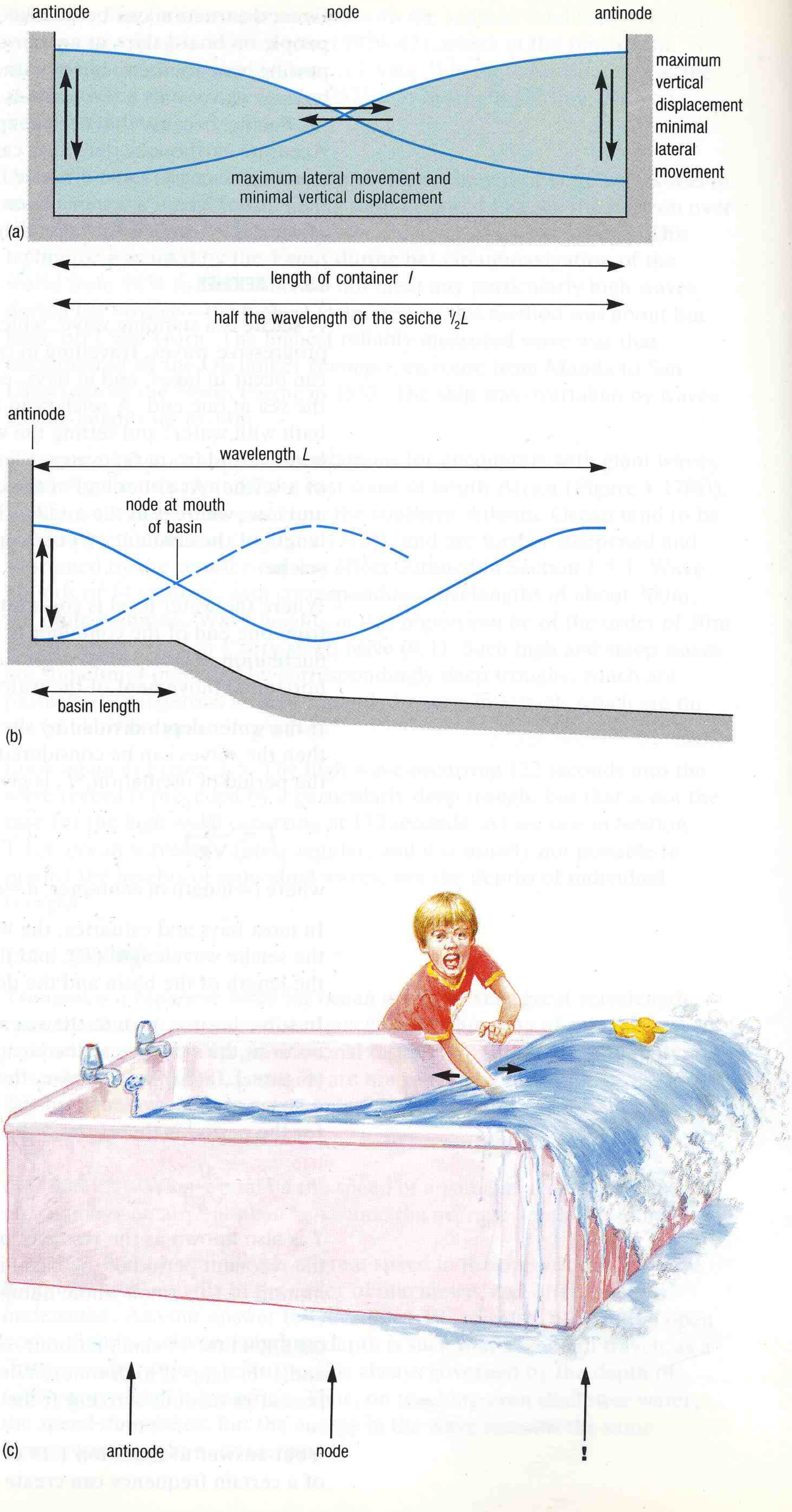


Figure 1.18(a) A simple standing wave.
(b) A quarter-wavelength standing wave in a small harbour.
(c) The hazards resulting from a seiche.

harbours by setting up a standing wave. Just as the seiche in a bath can be made to 'slop over' if you persist in wave generation for too long (Figure 1.18(c)), so a standing wave in a harbour may dash vessels against the harbour wall, or even throw them ashore. When the standing wave is at the low point of the antinode, there is also the danger of vessels being grounded, thus suffering damage to their hulls.

1.6 MEASUREMENT OF WAVES

Various instruments have been devised to measure wave characteristics. For example, a pressure gauge can be placed upon the sea-floor and will detect the frequency and size of waves passing above it. Pressure gauges with a sensitivity of about one part in one million are available. Such gauges can detect a water level change of less than a centimetre in a depth of several thousand metres of water. Another method is to place an instrument akin to an accelerometer in a moored buoy which then detects the rise and fall of waves displacing the buoy. Most wave measurement employs one or other of these methods.

1.6.1 SATELLITE OBSERVATIONS OF WAVES

The observation of ocean surface waves is particularly amenable to the use of satellite-based remote-sensing. Four main techniques are considered here, but the technical details are not important in the context of this Chapter:

1 Radar altimetry

A radar altimeter, mounted in a satellite, emits radar pulses at a rate of about 1000 pulses per second directly towards the sea-surface, and each reflected pulse is picked up by a sensor. Radar pulses pass through, and are not reflected by, water vapour in the atmosphere, but are reflected by the sea-surface. Two things are measured:

(i) The average time which the pulses take to travel to the sea-surface and back again, which enables the mean satellite-to-surface distance to be calculated. The accuracy with which the distance can be measured depends upon the sea state. For values of $H_{1/3}$ up to 8m, the accuracy is $\pm$ 8cm, and for values of $H_{1/3}$ over 8m, the accuracy is about $\pm\, x$cm, where x is the significant wave height in metres.

(ii) Changes in the shape and amplitude of the returning pulses, which can be used to give a measure of wave heights, from which the significant wave height ($H_{1/3}$) can be calculated. The accuracy achieved is about $\pm$10% of $H_{1/3}$ for waves of significant wave height greater than 3m, and about $\pm$0.3m for smaller values of $H_{1/3}$.

QUESTION 1.20 Estimate the uncertainties which would be expected in (i) the mean satellite-to-surface distance, and (ii) the significant wave height, if they were measured by radar altimeter under the following conditions:

(a) Force 1 wind on the Beaufort Scale, no cloud;
(b) Force 4 wind on the Beaufort Scale, dense cloud cover;
(c) Force 10 wind on the Beaufort Scale, light cloud cover.

Use Table 1.1, and assume a steady sea state has been established in each case.

2 Synthetic aperture radar (SAR)

The procedure maps variations in the proportion of the radar signal that is back-scattered to the radar antenna by the sea-surface, and gives a measure of its 'roughness'. The technique involves the transmission of short radar pulses at an oblique angle to the sea-surface, and analysing the Doppler shift (i.e. apparent changes in signal frequency because of relative movement between the target and the detector) of the scattered return signal in order to produce an image.

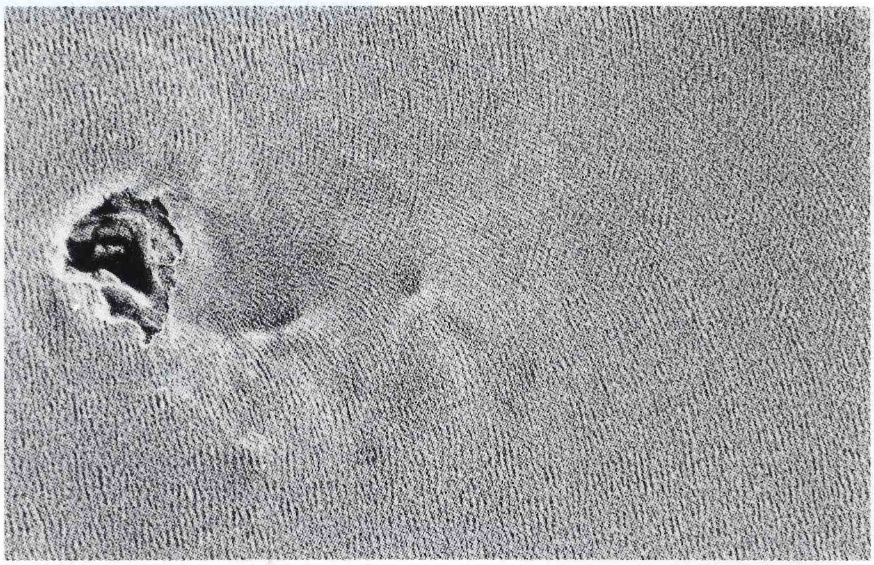


Figure 1.19 Swell-wave pattern around the island of Foula (70 km NW of Fair Isle, NE Atlantic) obtained by synthetic aperture radar (SAR). Area ≈ 30 km × 19 km.

SAR images often show a wave-like pattern (Figure 1.19), but this pattern is not a direct indication of wavelength. The regular variations in reflectivity can, however, be related to the characteristics of larger (swell) waves from which an idea of their wavelength and direction of propagation can be obtained. The precise interpretation of SAR images is hampered by such difficulties as:

(i) SAR assumes a moving satellite and a stationary target, whereas both waves and water may be moving. This motion complicates the return signal, so that elaborate procedures have to be applied to render the image into an interpretable form.

(ii) Image 'roughness' is affected by the entire spectrum of waves present, not just by the swell waves. Even capillary waves of a few millimetres wavelength influence the image, and hence complicate its interpretation.

(iii) White-capping tends to scatter incident radar randomly, thus obscuring the more regular back-scatter obtained from smoother surfaces. If waves actually break, the foam causes a significant absorption of the radar signals, reducing both clarity and contrast in the final image.

3 Scatterometry

Radar scatterometers are used to analyse the strength and polarization of radar echoes, from which surface wind strength and direction can be determined. The method is complex and relies on the different back-scattering properties of wind-generated surface waves when viewed obliquely in upwind, downwind and crosswind directions.

4 Photography
Changes in the amount of reflected sunlight correlate with local roughness and wave steepness, thus revealing wave patterns on photographs. Even internal waves can be detected by photography because of their effect on surface roughness. Figure 1.20 was taken from a manned spacecraft, with a hand-held camera, and shows some internal waves in the South China Sea.

Figure 1.20(a) Internal waves in the South China Sea (Hainan Island visible beneath clouds on lower left). Four wave packets are visible.
(b) Tidally generated internal waves propagating into the Mediterranean from the Straits of Gibraltar. The internal waves which have amplitudes of the order of 50 feet are visible because of variations in the roughness of the sea-surface. Area = 74 km × 74 km.

In spite of the difficulties and expense, remote-sensing techniques provide a unique elevated view of the wide expanses of the oceans, and offer the only practical method of obtaining the widespread and repetitive observations that are necessary if reliable predictions of the behaviour of the sea-surface are to be made.

1.7 SUMMARY OF CHAPTER 1

1 Idealized waves of sinusoidal form have wavelength (length between successive crests), height (vertical difference between trough and crest), steepness (ratio of height to length), amplitude (half the wave height), period (length of time between successive waves passing a fixed point) and frequency (reciprocal of period). Waves transfer energy across material without significant *overall* motion of the material itself, but individual particles are displaced from, and return to, equilibrium positions as each wave passes. Surface waves occur at interfaces between fluids, either because of relative movement between them, or because the fluids are disturbed by an external force. Waves occurring at interfaces between oceanic water layers are called internal waves. Water waves, once initiated, are maintained by surface tension and gravitational force, although only the latter is significant for water waves over 1.7cm wavelength.

2 Most sea-surface waves are wind-generated. They obtain energy from pressure differences resulting from the sheltering effect provided by the wave crests. The stronger the wind, the larger the wave, so variable winds produce a range of wave sizes. A constant wind speed produces a fully developed sea, with waves of $H_{1/3}$ (average height of highest 33% of the waves) characteristic of that wind speed. The Beaufort Scale relates sea state and $H_{1/3}$ with the causative wind speed. Water waves show cyclical variations in water level (displacement), from $-a$ (amplitude) in the trough to $+a$ at the crest. Displacement varies not only in space (one wavelength between successive crests) but also in time (one period between successive crests at one location). Steeper waves depart from the simple sinusoidal model, and more closely resemble a trochoidal wave-form.

3 Water particles in waves over deep water follow almost circular paths, but with a small net forward drift. Path diameters at the surface correspond to wave heights, but decrease exponentially with depth. In shallow water, the orbits become flattened near the sea-bed. Wave speed equals wavelength/period (or angular frequency/wave number), and is influenced both by wavelength and by depth. However, for waves in water deeper than 1/2 wavelength, wave speed is proportional to the square root of the wavelength, and is unaffected by depth. For waves in water shallower than 1/20 wavelength, wave speed is proportional to the square root of the depth, and does not depend upon the wavelength. For idealized water waves, the three characteristics c, L, and T are related by the equation $c = L/T$. In addition, each can be expressed in terms of each of the other two. For example $c = 1.56T$ and $L = 1.56T^2$.

4 Waves of different sizes become dispersed, because those with greater wavelengths and longer periods travel faster than smaller waves. If two wave trains of similar wavelength and amplitude travel over the same sea area, they interact. Where they are in phase, displacement is doubled, whereas where they are out of phase, displacement is extinguished. A single wave train results, travelling as a series of wave groups, each separated from adjacent groups by an almost wave-free region. Wave group speed in deep water is half the average speed of the two component wave trains. In shallowing water, wave speed approaches group speed, until the two coincide at depths less than 1/20 wavelength.

5 Wave energy is proportional to wave height squared, and is propagated at group speed. Total energy is conserved in a given length of wave crest, so waves entering shallowing water increase in height as their group speed falls. Wave power is wave energy propagated per second per unit length of wave crest (or wave speed multiplied by wave energy per unit area). Wave energy has been successfully harnessed on a small scale, but large scale utilization involves some environmental and navigational problems, and huge capital outlay.

6 Dissipation of wave energy (attenuation of waves) results from white-capping, friction between water molecules, air resistance, and non-linear wave–wave interaction (exchange of energy between waves of differing frequencies). Swell waves are storm-generated, travel far from their place of origin, and once established are little affected by wind or by smaller waves. Most attenuation takes place in the storm area.

7 Waves in shallow water may be refracted. Variations in depth cause variations in speed of different parts of the wave crest, which as a result

becomes refracted so as to trend parallel with bottom contours. The energy of refracted waves is conserved, so converging waves tend to increase, and diverging waves to diminish, in height. Breakers dissipate wave energy. In general, the steeper the wave and the shallower the beach, the further offshore dissipation begins. Breakers form a continuous series from steep spilling types to long-period surging breakers.

8 Waves propagating with a current have diminished heights, whereas a counter-current increases wave height, unless current speed exceeds half the wave group speed. If so, waves no longer propagate, but increase in height until they become unstable and break. Tsunamis are caused by earthquakes or by slumping of sediments, and their great wavelength means their speed is always governed by the ocean depth. Wave height is small in the open ocean, but can become destructively large near the shore. Seiches (standing waves) oscillate, so that at the antinodes there are great extremes of water level, but little lateral water movement, whereas at nodes the converse is true. The period of oscillation is proportional to container length and inversely proportional to the square root of the depth. A seiche is readily established when container length is a simple multiple of one-quarter of the seiche wavelength.

9 Waves are measured by a variety of methods, e.g. pressure gauges on the sea-floor, accelerometers in buoys on the sea-surface, and remote-sensing from satellites.

Now try the following questions to consolidate your understanding of this Chapter.

QUESTION 1.21 A wave of period 10 seconds approaching the shore has a height of 1m in deep water. Calculate:

(a) the wave speed and group speed in deep water;

(b) the wave steepness in deep water;

(c) the wave power per metre of crest in deep water;

(d) the wave power per metre of crest in water 2.5m deep.

QUESTION 1.22 A wave system consisting of short waves (wavelength 6m), together with a swell of period 22s, propagates through a narrow inlet, in which a current of 3 knots ($1.54\,\mathrm{m\,s^{-1}}$) runs, counter to the direction of wave propagation. Describe the wave characteristics:

(a) in the narrow inlet;

(b) at a point where the waves have passed beyond the narrow inlet into a region where the current is negligible.

(Assume the water is very deep at all the locations described.)

QUESTION 1.23 The *Ramapo* (refer back to report of waves 34m in height, Section 1.5.2) was a tanker 146m long. Assume the ship was steaming at a reduced speed of 10 knots ($5.14\,\mathrm{m\,s^{-1}}$), and that the wave crests took 6.3s to pass the ship from stern to bow.

(a) What was the wave speed?

(b) What was the wave steepness?

(c) How does the wave period, consistent with the answers to (a) and (b), compare with the wave period of 14.8s reported by the *Ramapo*?

(d) What would be the implications for maximum wave steepness of a wave period of 14.8s?

(e) Comment upon any inconsistencies revealed.

QUESTION 1.24 What sort of waves would you expect to see on a beach of intermediate slope:

(a) after a prolonged spell of calm weather?

(b) during a severe gale, with a Force 9 wind blowing onshore?