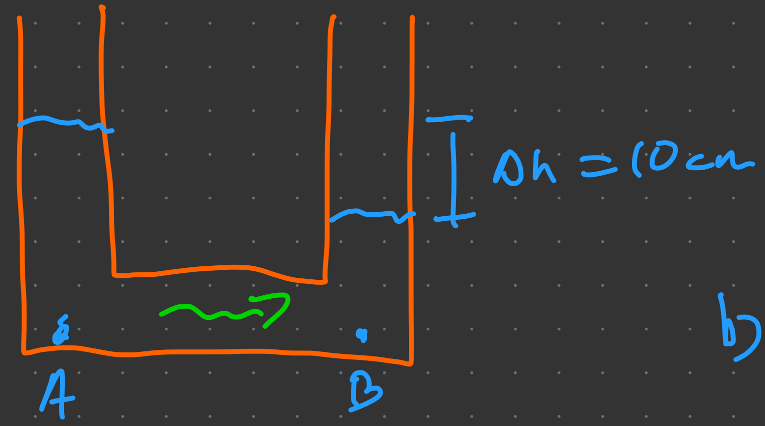
Assign 2, Q 1



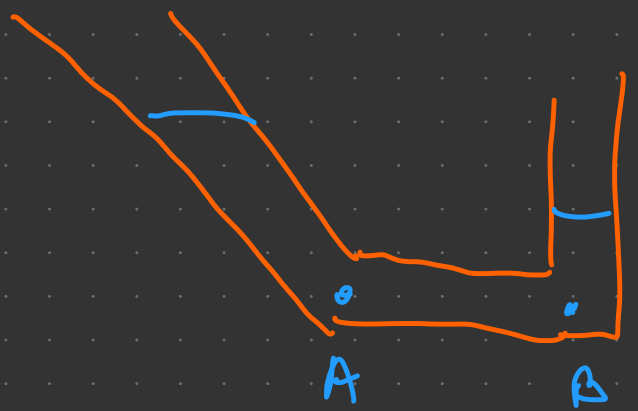
a) $\Delta P = \rho g h_A - \rho g h_B$

$= \rho g \, 0.1\,m$

$= 980 \; N/m^2$

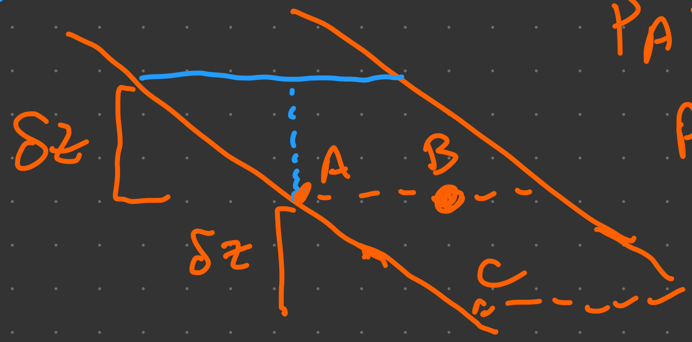
b) flow from left to right

c)



c) P_A does not care about the bend.

– why? At any depth pressure is the same so



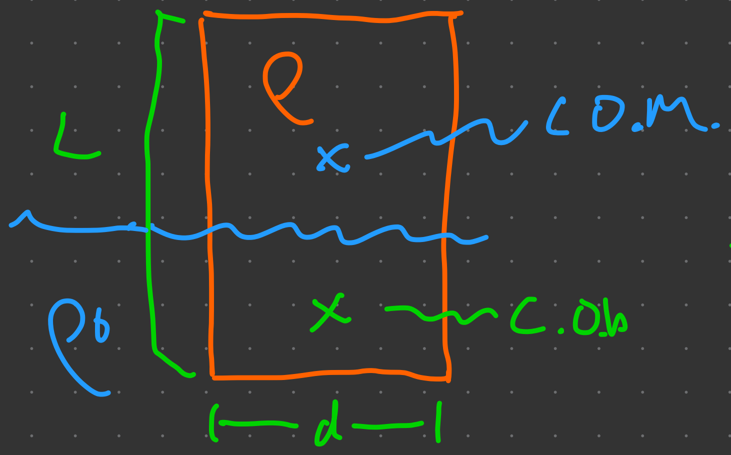
$P_A = \rho g \delta z = P_B$

$P_C = P_B + \rho g \delta z$

which is same as if it were not bent

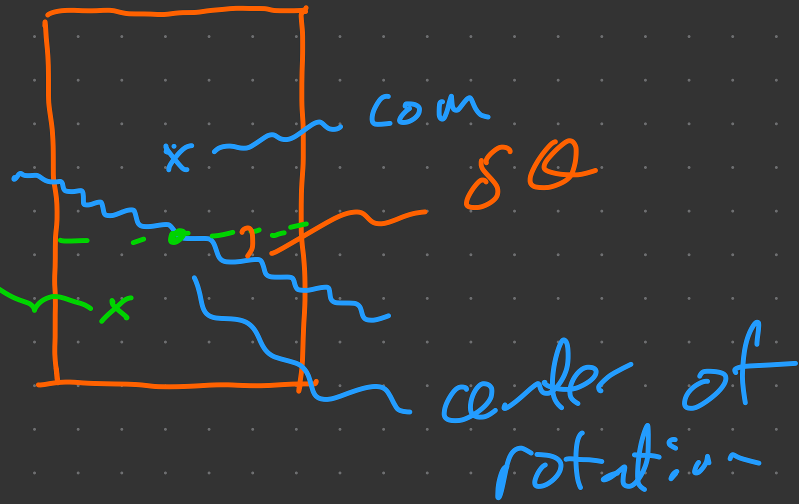
Q2

$-\ \rho_0$ density water, ρ density block



$$= L\frac{\rho}{\rho_0}$$

Now tilt small $\delta\theta$ and change co-ordinate system



In tilted frame note

$$\underline{F}_{com} = \rho g L d(-\delta\theta, -1)$$

$$\underline{F}_{cob} = \rho g L d(\delta\theta, 1)$$

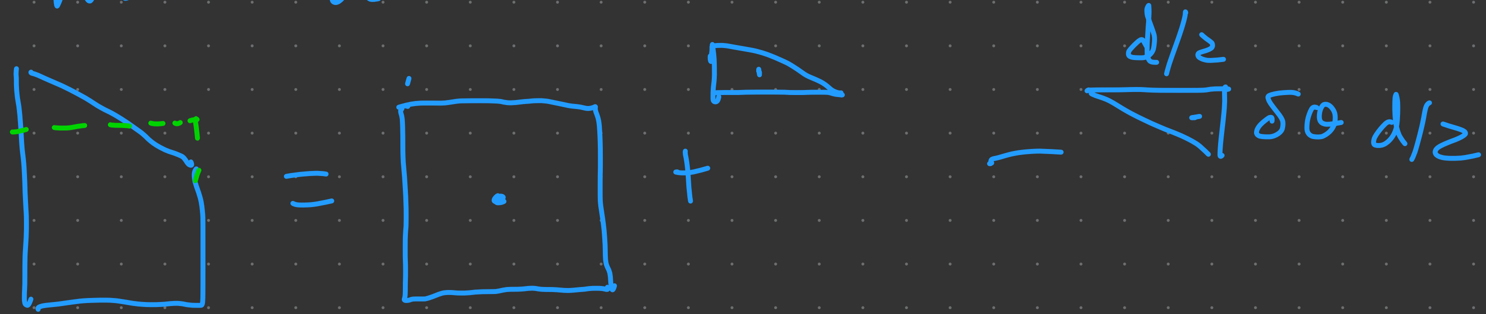
The idea is to get the torque around the centre of rotation, so we need the distance of the center of mass and center of buoyancy from the center of rotation:

$$\underline{r}_{com} = \left(0, \frac{L}{2} - L\frac{\rho}{\rho_0}\right) = \left(0, L\left(\frac{1}{2} - \frac{\rho}{\rho_0}\right)\right)$$

So the torque c.o.m is just $\underset{\sim}{r} \times \underset{\sim}{F}$

$$\tau_{com} = xF_y - yF_x = \rho g L d^2 \left(\frac{1}{2} - \frac{\rho}{\rho_0}\right) \delta\theta$$

Now, the centre of buoyancy is harder because it moves. Note that



$$A_0 \underset{\sim}{r}_{COB} = A_0 \left(0, \frac{L\rho}{2\rho_0}\right) + \left(-\frac{2}{3}\frac{d}{2}, \frac{\delta\theta}{3}\frac{d}{2}\right) A_T$$

$$- \left(\frac{2}{3}\frac{d}{2}, -\frac{\delta\theta}{3}\frac{d}{2}\right) A_T$$

$$A_0 = d\,\frac{L\rho}{2\rho_0} \;;\quad A_T = \frac{1}{2}\frac{d^2}{4}\delta\theta \;;\quad A_T/A_0 = \frac{d}{4}\frac{\rho_0}{\rho}\delta\theta$$

$$\underset{\sim}{r}_{COB} = \left(-\frac{1}{12}\frac{d^2\,\delta\theta}{w},\ -\frac{w}{2} \right)$$ dropping $\delta\theta^2$ terms

So $$\underset{\sim}{\tau}_{COB} = \rho g L d \left(-\frac{1}{12}\frac{d^2}{w}\delta\theta + \frac{w}{2}\delta\theta \right)$$

$\tau_{COM} + \tau_{COB} < 0$ if stable
(ie fixed $\delta\theta$)

$$\Rightarrow L\left(\frac{1}{2} - \frac{\rho}{\rho_0}\right) + \frac{w}{2} - \frac{d^2}{12w} < 0$$

$$\Rightarrow \boxed{6\left(1 - \frac{\rho}{\rho_0}\right)\frac{\rho}{\rho_0} < d^2/L^2}$$

if stable!