Take-home Final Assignment
Phy 426, 2026
J. Klymak

DUE: 15 Apr, 2026, 14:00

Exam to be completed independently, though questions to the instructor are welcome/encouraged. Open-book, open-notes are fine. Show all work, define any constants you need that I don't provide, check your units, etc. Except as noted, the density of the fluid is ρ, gravity is g, the kinematic viscosity ν, and the fluid can be assumed Bousinesque and incompressible.

Please try to make it readable. I will deduct up to 10% for illegible chicken scratches, so please take the time to recopy your work. Typeset is preferred but not necessary.

Question 1. Shear production of turbulent kinetic energy

Consider a slowly evolving flow in the x-direction with a slowly evolving shear in the z-direction given by $\partial U/\partial z \neq 0$. On top of this slowly evolving flow is a flow that is turbulent with velocity fluctuations u', v', w'

1. [10] Using Reynolds averaging form an energy equation for the mean flow and show that the mean flow has a sink of energy given by $+\overline{u'w'}\frac{\partial U}{\partial z}$
2. [4] Explain why this is a sink instead of a source by considering the likely sign of the terms.
3. [8] Again using Reynolds averaging on the energy equation, show that the *turbulent* kinetic energy has a *source* due to this term.

Question 2. Oscillating forcing of a fluid above a plate

Consider a fluid with viscosity ν that has an infinite plate as a no-slip bottom boundary condition. The fluid is infinite in y above the plate. Suppose that the fluid is everywhere subject to an oscillating horizontal pressure gradient (parallel to the plate), given by

$$\frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}x} = A\cos\omega t \qquad (1)$$

where ω is the forcing frequency in rad/s.

1. [15] Show that a solution for the flow as a function of y and t, assuming the flow is well-developed (e.g. long after any startup transient) is given by:

   $$u(y,t) = -\frac{A}{\omega}\left(\sin\omega t\left(1 - e^{-ky}\cos ky\right) + \cos\omega t\ e^{-ky}\sin ky\right) \qquad (2)$$

   where $k \equiv \sqrt{\frac{\omega}{2\nu}}$.

   Be sure to describe the steps you take to solve this problem, and to clearly state any assumptions you make.

2. [5] Comment on the character of the solution, particularly with respects to the phase of the solution both far from and near the plate. A sketch is helpful - and while you are not required to, I found doing this quickly in a plotting package helpful.

3. [10] Calculate the work per length x done by the pressure force on the flow, and show that it is equal to the viscous dissipation in the flow, averaged over a forcing cycle.

Question 3. Two-layer internal tide generation at a step

Consider a two-layer fluid with a step in the bottom topography. The upper layer has resting depth h_1 and density ρ_1, and the lower layer has a resting depth h_2 and density ρ_2. The density difference is small compared to the absolute density, so $\Delta\rho = \rho_2 - \rho_1 \ll \rho_1, \rho_2$. The bottom topography is $z = -h_1 - h_2$ for $x < 0$ and $z = -h_1 - h_2 + b$ for $x > 0$ if the seasurface is at $z = 0$.

A tide of frequency ω comes from the left ($x < 0$) direction with a seasurface amplitude $\eta_I = A_I \cos\left(k_{FL}(x - c_{FL}t)\right)$, where $c_{FL} = \sqrt{g(h_1 + h_2)}$ is the *Fast* surface wave speed, and $k_{FL} = \omega/c_{FL}$ is the wavenumber of the incoming tide.

1. [5] Sketch the problem and indicate the waves you expect to see generated by this flow.

2. [10] Solve for the amplitude of the reflected and transmitted surface waves under the assumption that the surface response does not know about the internal waves and argue that

$$A_R \approx \frac{c_{FR} - c_{FL}}{c_{FR} + c_{FL}} A_I \tag{3}$$

and

$$A_T \approx \frac{2c_{FR}}{c_{FR} + c_{FL}} A_I \tag{4}$$

where $c_{FL} = \sqrt{g(h_1 + h_2)}$ and $c_{FR} = \sqrt{g(h_1 + h_2 - b)}$ are the fast surface wave speeds on the left and right sides of the step, respectively. (Hint: you will need two boundary conditions at $x = 0$; one is that the sea surface elevation is continuous, and the other is that the transport of water across the boundary is continuous).

3. [20] At the step ($x = 0$), let the total interface displacement on the left and right be

$$\zeta^-(0) = \zeta_I^s + \zeta_R^s + \zeta_L^i \tag{5}$$

and

$$\zeta^+(0) = \zeta_T^s + \zeta_R^i \tag{6}$$

where ζ_I^s, ζ_R^s, and ζ_T^s are the interface displacements induced by incoming, reflected, and transmitted *surface* waves, and ζ_L^i and ζ_R^i are the generated *internal* waves radiating to the left and right, respectively.

The first boundary condition is continuity of interface displacement:

$$\zeta^-(0) = \zeta^+(0) \tag{7}$$

with $\zeta_I^s = \alpha_L A_I$, $\zeta_R^s = \alpha_L A_R$, and $\zeta_T^s = \alpha_R A_T$.

The second boundary condition is continuity of lower-layer transport at $x = 0$:

$$Q_2^-(0) = Q_2^+(0) \tag{8}$$

with

$$Q_2^-(0) = h_2 \left(u_{2I} - u_{2R} + u_{2L}^i\right), \qquad Q_2^+(0) = (h_2 - b)\left(u_{2T} + u_{2R}^i\right) \tag{9}$$

where the minus sign on u_{2R} accounts for leftward propagation of the reflected wave. Use these two conditions to solve for

$$\zeta_L^i = \frac{S + c_{IR}\Delta}{c_{IL} + c_{IR}} \tag{10}$$

and

$$\zeta_R^i = \frac{S - c_{IL}\Delta}{c_{IL} + c_{IR}} \tag{11}$$

where

$$\Delta = \alpha_R A_T - \alpha_L A_I - \alpha_L A_R \tag{12}$$

and

$$S = \frac{h_2}{h_1 + h_2}(A_I - A_R)\, c_{FL} - \frac{h_2 - b}{h_1 + h_2 - b} A_T c_{FR} \tag{13}$$

and $\alpha_L = \frac{h_2}{h_1+h_2}$, $\alpha_R = \frac{h_2-b}{h_1+h_2-b}$ are geometric parameters, $c_{IL} = \sqrt{g' \frac{h_1 h_2}{h_1+h_2}}$, and $c_{IR} = \sqrt{g' \frac{(h_1)(h_2-b)}{h_1+h_2-b}}$ are the internal wave speeds on the left and right sides of the step, respectively.

4. [10] So lets check if this all makes sense: Suppose $\Delta\rho = 1\text{kg/m}^3$, $h_1 = 20\text{m}$, $h_2 = 100\text{m}$, $b = 50\text{m}$, and $A_I = 1\text{m}$. Calculate the amplitudes of the reflected and transmitted surface waves, and the internal waves radiating to the left and right. *Comment* on the relative sizes of these waves compared to the surface forcing.

5. [15] Given the amplitudes of the internal waves, calculate the energy flux of the internal waves radiating to the left and right, and compare it to the energy flux of the incoming surface tide. *Comment* on the *efficiency* of internal tide generation at this step (eg what fraction of surface energy goes into internal tidal energy) (Hint: the energy flux of a wave is given by Ec_g where E is the energy density of the wave and c_g is the group velocity of the wave. The energy density of a surface wave is given by $E = \frac{1}{2}\rho g A^2$, and the energy density of an internal wave is given by $E = \frac{1}{2} g' \zeta^2$, and recall there is one radiating in each direction)

Answer Key for Exam A

Question 1. Shear production of turbulent kinetic energy

Consider a slowly evolving flow in the x-direction with a slowly evolving shear in the z-direction given by $\partial U/\partial z \neq 0$. On top of this slowly evolving flow is a flow that is turbulent with velocity fluctuations u', v', w'

1. [10] Using Reynolds averaging form an energy equation for the mean flow and show that the mean flow has a sink of energy given by $+\overline{u'w'}\frac{\partial U}{\partial z}$

   Answer: This is covered explicitly in the text in the chapter on turbulence: "Kinetic Energy Budget of Mean Flow".

2. [4] Explain why this is a sink instead of a source by considering the likely sign of the terms.

   Answer: This is again covered explicitly in the text: if $dU/dz > 0$ then $w' > 0$ will, on average bring water upwards so that it is moving slower then U, and hence $u' < 0$. Similarly if the converse is true, and hence $\overline{u'w'} < 0$, and $\overline{u'w'}\frac{\partial U}{\partial z} < 0$. The converse all applies if $dU/dz < 0$.

3. [8] Again using Reynolds averaging on the energy equation, show that the *turbulent* kinetic energy has a *source* due to this term.

   Answer: This is covered in the *next* section, entitled "Kinetic Energy Budget of Turbulent Flow" where after all is done, the term $-\overline{u'w'}\frac{\partial U}{\partial z}$ appears, and is hence usually a source.

Question 2. Oscillating forcing of a fluid above a plate

Consider a fluid with viscosity ν that has an infinite plate as a no-slip bottom boundary condition. The fluid is infinite in y above the plate. Suppose that the fluid is everywhere subject to an oscillating horizontal pressure gradient (parallel to the plate), given by

$$\frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}x} = A\cos\omega t \tag{1}$$

where ω is the forcing frequency in rad/s.

1. [15] Show that a solution for the flow as a function of y and t, assuming the flow is well-developed (e.g. long after any startup transient) is given by:

   $$u(y,t) = -\frac{A}{\omega}\left(\sin\omega t\left(1 - e^{-ky}\cos ky\right) + \cos\omega t\; e^{-ky}\sin ky\right) \tag{2}$$

   where $k \equiv \sqrt{\frac{\omega}{2\nu}}$.

   Be sure to describe the steps you take to solve this problem, and to clearly state any assumptions you make.

Answer: The flow is subject to the equation

$$\frac{\partial u_0}{\partial t} = -A\cos\omega t + \nu\frac{\partial^2 u_0}{\partial y^2} \tag{3}$$

We can write as a complex differential equation, with $u_0 = \mathcal{R}(u)$:

$$\frac{\partial u}{\partial t} = -Ae^{i\omega t} + \nu\frac{\partial^2 u}{\partial y^2} \tag{4}$$

where $u(0,t) = 0$ and $\frac{\partial u}{\partial y} \to 0$ as $y \to \infty$ The solution will oscillate at the same frequency, though not necessarily in phase, and it is a forced oscillator, so there is a component to the response in balance with the forcing: $u(y,t) = \left(U(y) + \frac{A\,i}{\omega}\right)e^{i\omega t}$, leaving a differential equation

$$i\omega U = \nu\frac{\partial^2 U}{\partial y^2} \tag{5}$$

This has combined oscillatory/decaying solutions in y

$$U(y) = be^{\pm ky}e^{\pm iky} \tag{6}$$

where $k = \sqrt{\frac{\omega}{2\nu}}$. The boundary conditions on $U(y)$ preclude the exponentially growing solution, so the result is

$$U(y) = be^{-ky}e^{-iky} \tag{7}$$

The boundary condition at $y = 0$ gives

$$u(y,t) = \frac{A\,i}{\omega}\left(1 - e^{-ky}e^{-iky}\right)e^{i\omega t} \tag{8}$$

which gives, for the real part:

$$u_0(y,t) = -\frac{A}{\omega}\left(\sin\omega t\left(1 - e^{-ky}\cos ky\right) + \cos\omega t\ e^{-ky}\sin ky\right) \tag{9}$$

2. [5] Comment on the character of the solution, particularly with respects to the phase of the solution both far from and near the plate. A sketch is helpful - and while you are not required to, I found doing this quickly in a plotting package helpful.

Answer: Far from the plate, the velocity is exactly 90 degrees out of phase with the pressure forcing, as expected.

Close to the plate, the velocity is of course attenuated. However, it also *leads* the velocity far from the plate. This is perhaps unsurprising as the pressure gradient is the same as aloft, but the flow speed smaller so the flow reverses direction sooner.

3. [10] Calculate the work per length x done by the pressure force on the flow, and show that it is equal to the viscous dissipation in the flow, averaged over a forcing cycle.

Answer: The pressure work done, averaged over the forcing cycle, is

$$W = \frac{1}{T}\int_0^T \mathrm{d}t \int_0^\infty -\frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}x}u\ \mathrm{d}y \tag{10}$$

where $T = \frac{2\pi}{\omega}$.

$$W = \frac{1}{T}\int_0^T \mathrm{d}t \int_0^\infty \frac{A^2}{\omega}\left(\cos\omega t \sin\omega t\left(1 - e^{-ky}\cos ky\right) + \cos^2(\omega t)\, e^{-ky}\sin ky\right)\mathrm{d}y \quad (11)$$

The first term integrates in time to zero, and the second to:

$$W = \frac{A^2}{2\omega}\int_0^\infty e^{-ky}\sin ky\ \mathrm{d}y \quad (12)$$

or

$$W = \frac{A^2}{4\omega k} \quad (13)$$

For the dissipation

$$D = \frac{1}{T}\int_0^T \mathrm{d}t \int_0^\infty \nu\left(\frac{\partial u}{\partial y}\right)^2\ \mathrm{d}y \quad (14)$$

This is much easier to solve as a complex velocity and then take the real part $D = \mathcal{R}D'$:

$$D' = \frac{1}{T}\int_0^T \mathrm{d}t \int_0^\infty \nu\left(\frac{Aki}{\omega}(1+i)\,e^{-k(1+i)}e^{i\omega t}\right)^2\ \mathrm{d}y \quad (15)$$

$$D' = \frac{1}{T}\int_0^T \mathrm{d}t \int_0^\infty \nu 2i\frac{A^2k^2}{\omega^2}e^{i2\omega t}e^{-2ky(1+i)}\ \mathrm{d}y \quad (16)$$

These are both easy integrals, and the real part yields:

$$D = \nu\frac{kA^2}{2\omega^2} \quad (17)$$

and recalling that $\nu = \frac{\omega}{2k^2}$

$$D = \frac{A^2}{4k\omega} \quad (18)$$

Question 3. Two-layer internal tide generation at a step

Consider a two-layer fluid with a step in the bottom topography. The upper layer has resting depth h_1 and density ρ_1, and the lower layer has a resting depth h_2 and density ρ_2. The density difference is small compared to the absolute density, so $\Delta\rho = \rho_2 - \rho_1 \ll \rho_1, \rho_2$. The bottom topography is $z = -h_1 - h_2$ for $x < 0$ and $z = -h_1 - h_2 + b$ for $x > 0$ if the seasurface is at $z = 0$.

A tide of frequency ω comes from the left ($x < 0$) direction with a seasurface amplitude $\eta_I = A_I \cos(k_{FL}(x - c_{FL}t))$, where $c_{FL} = \sqrt{g(h_1 + h_2)}$ is the *Fast* surface wave speed, and $k_{FL} = \omega/c_{FL}$ is the wavenumber of the incoming tide.

1. [5] Sketch the problem and indicate the waves you expect to see generated by this flow.

Answer: The incoming tide will generate a reflected surface wave, a transmitted surface wave, and an internal wave that propagates away from the step in both directions.

2. [10] Solve for the amplitude of the reflected and transmitted surface waves under the assumption that the surface response does not know about the internal waves and argue that

$$A_R \approx \frac{c_{FR} - c_{FL}}{c_{FR} + c_{FL}} A_I \tag{19}$$

and

$$A_T \approx \frac{2c_{FR}}{c_{FR} + c_{FL}} A_I \tag{20}$$

where $c_{FL} = \sqrt{g(h_1 + h_2)}$ and $c_{FR} = \sqrt{g(h_1 + h_2 - b)}$ are the fast surface wave speeds on the left and right sides of the step, respectively. (Hint: you will need two boundary conditions at $x = 0$; one is that the sea surface elevation is continuous, and the other is that the transport of water across the boundary is continuous).

Answer: This is a relatively straight forward scattering problem. The condition at $x = 0$ is that the surface elevation and the transport of water across the boundary must be continuous.

The reflected wave has the form:

$$\eta_R = A_R \cos k_{FL}(x + c_{FL}t) \tag{21}$$

and the transmitted wave has the form

$$\eta_T = A_T \cos k_{FL}(x - c_{FL}t) \tag{22}$$

The first condition gives

$$A_I + A_R = A_T \tag{23}$$

The second condition is $u_L\,(h_1 + h_2) = u_R\,(h_1 + h_2 - b)$, where u_L and u_R are the horizontal velocities at $x = 0$ on the left and right sides of the step. From the surface wave momentum equations we know that $\hat{u}h = c\hat{\eta}$, so the second condition gives:

$$A_I + A_R = \frac{c_{FR}}{c_{FL}} A_T \tag{24}$$

and we can solve for the reflected and transmitted wave amplitudes:

$$A_R = \frac{c_{FR} - c_{FL}}{c_{FR} + c_{FL}} A_I \tag{25}$$

$$A_T = \frac{2c_{FR}}{c_{FR} + c_{FL}} A_I \tag{26}$$

3. [20] At the step ($x = 0$), let the total interface displacement on the left and right be

$$\zeta^-(0) = \zeta_I^s + \zeta_R^s + \zeta_L^i \tag{27}$$

and

$$\zeta^+(0) = \zeta_T^s + \zeta_R^i \tag{28}$$

where ζ_I^s, ζ_R^s, and ζ_T^s are the interface displacements induced by incoming, reflected, and transmitted *surface* waves, and ζ_L^i and ζ_R^i are the generated *internal* waves radiating to the left and right, respectively.

The first boundary condition is continuity of interface displacement:

$$\zeta^-(0) = \zeta^+(0) \tag{29}$$

with $\zeta_I^s = \alpha_L A_I$, $\zeta_R^s = \alpha_L A_R$, and $\zeta_T^s = \alpha_R A_T$.
The second boundary condition is continuity of lower-layer transport at $x = 0$:

$$Q_2^-(0) = Q_2^+(0) \tag{30}$$

with

$$Q_2^-(0) = h_2 \left(u_{2I} - u_{2R} + u_{2L}^i\right), \qquad Q_2^+(0) = (h_2 - b)\left(u_{2T} + u_{2R}^i\right) \tag{31}$$

where the minus sign on u_{2R} accounts for leftward propagation of the reflected wave.
Use these two conditions to solve for

$$\zeta_L^i = \frac{S + c_{IR}\Delta}{c_{IL} + c_{IR}} \tag{32}$$

and

$$\zeta_R^i = \frac{S - c_{IL}\Delta}{c_{IL} + c_{IR}} \tag{33}$$

where

$$\Delta = \alpha_R A_T - \alpha_L A_I - \alpha_L A_R \tag{34}$$

and

$$S = \frac{h_2}{h_1 + h_2}\left(A_I - A_R\right) c_{FL} - \frac{h_2 - b}{h_1 + h_2 - b} A_T c_{FR} \tag{35}$$

and $\alpha_L = \frac{h_2}{h_1+h_2}$, $\alpha_R = \frac{h_2-b}{h_1+h_2-b}$ are geometric parameters, $c_{IL} = \sqrt{g' \frac{h_1 h_2}{h_1+h_2}}$, and $c_{IR} = \sqrt{g' \frac{(h_1)(h_2-b)}{h_1+h_2-b}}$ are the internal wave speeds on the left and right sides of the step, respectively.

Answer: Using continuity of interface displacement,

$$\zeta_I^s + \zeta_R^s + \zeta_L^i = \zeta_T^s + \zeta_R^i \tag{36}$$

and therefore

$$\zeta_L^i - \zeta_R^i = \alpha_R A_T - \alpha_L A_I - \alpha_L A_R \equiv \Delta \tag{37}$$

with $\zeta_I^s = \alpha_L A_I$, $\zeta_R^s = \alpha_L A_R$, and $\zeta_T^s = \alpha_R A_T$.
For lower-layer transport continuity,

$$h_2 \left(u_{2I} - u_{2R} + u_{2L}^i\right) = (h_2 - b)\left(u_{2T} + u_{2R}^i\right) \tag{38}$$

where $u_{2I} = A_I \frac{c_{FL}}{h_1+h_2}$, $u_{2R} = A_R \frac{c_{FL}}{h_1+h_2}$, and $u_{2T} = A_T \frac{c_{FR}}{h_1+h_2-b}$.
The internal-wave velocity/displacement relations are

$$h_2 u_{2L}^i = -c_{IL}\zeta_L^i \tag{39}$$

$$(h_2 - b) u_{2R}^i = c_{IR}\zeta_R^i \tag{40}$$

where $c_{IL} = \sqrt{\frac{g\Delta\rho}{\rho}\frac{h_1 h_2}{h_1+h_2}}$ and $c_{IR} = \sqrt{\frac{g\Delta\rho}{\rho}\frac{(h_1)(h_2-b)}{h_1+h_2-b}}$.

Substituting gives

$$c_{IL}\zeta_L^i + c_{IR}\zeta_R^i = \frac{h_2}{h_1+h_2}\left(A_I - A_R\right)c_{FL} - \frac{h_2 - b}{h_1+h_2-b}A_T c_{FR} \equiv S \tag{41}$$

So we solve

$$\zeta_L^i - \zeta_R^i = \Delta \tag{42}$$

and

$$c_{IL}\zeta_L^i + c_{IR}\zeta_R^i = S \tag{43}$$

to obtain

$$\zeta_L^i = \frac{S + c_{IR}\Delta}{c_{IL}+c_{IR}} \tag{44}$$

and

$$\zeta_R^i = \frac{S - c_{IL}\Delta}{c_{IL}+c_{IR}} \tag{45}$$

4. [10] So lets check if this all makes sense: Suppose $\Delta\rho = 1\text{kg/m}^3$, $h_1 = 20\text{m}$, $h_2 = 100\text{m}$, $b = 50\text{m}$, and $A_I = 1\text{m}$. Calculate the amplitudes of the reflected and transmitted surface waves, and the internal waves radiating to the left and right. *Comment* on the relative sizes of these waves compared to the surface forcing.

Answer: Lets define some constants: $\alpha_L = 100/120 = 0.833$, $\alpha_R = 50/70 = 0.714$, $c_{FL} = \sqrt{9.81 * 120} = 34.3\text{m/s}$, $c_{FR} = \sqrt{9.81*70} = 26.2\text{m/s}$, $c_{IL} = \sqrt{9.81*1/1025*20*100/120} = 1.27\text{m/s}$, and $c_{IR} = \sqrt{9.81*1/1025*20*50/70} = 0.95\text{m/s}$.
From these we see that

$$A_R = \frac{26.2 - 34.3}{26.2+34.3} * 1 = -0.12\text{m} \tag{46}$$

and

$$A_T = \frac{2*26.2}{26.2+34.3} * 1 = 0.88\text{m} \tag{47}$$

Note that $\Delta = 0.714*0.88 - 0.833*1 - 0.833*(-0.12) = -0.05\text{m}$ and $S = 0.833*(1-(-0.12))*34.3 - 0.714*0.88*26.2 = 28.8\text{m}^2/\text{s}$, so
Now we can calculate the internal wave amplitudes:

$$\zeta_l = \frac{S + c_{IR}\Delta}{c_{IL}+c_{IR}} = \frac{28.8 + 0.95*(-0.05)}{1.27+0.95} = 13.9\text{m} \tag{48}$$

$$\zeta_r = \frac{S - c_{IL}\Delta}{c_{IL}+c_{IR}} = \frac{28.8 - 1.27*(-0.05)}{1.27+0.95} = 14.0\text{m} \tag{49}$$

So the internal wave amplitudes are much larger than the surface wave amplitudes, which is a common feature of internal tide generation. Note that they are also in phase with the incoming surface tide at the generation point ($x = 0$).

5. [15] Given the amplitudes of the internal waves, calculate the energy flux of the internal waves radiating to the left and right, and compare it to the energy flux of the incoming surface tide. *Comment* on the *efficiency* of internal tide generation at this step (eg what fraction of surface energy goes into internal tidal energy) (Hint: the energy flux of a wave is given by Ec_g where E is the energy density of the wave and c_g is the group velocity of the wave. The energy density of a surface wave is given by $E = \frac{1}{2}\rho g A^2$, and the energy density of an internal wave is given by $E = \frac{1}{2}g'\zeta^2$, and recall there is one radiating in each direction)

Answer: Incoming energy is $E_I c_{gI} = \frac{1}{2}\rho g A_I^2 c_{FL} = 168\mathrm{kW/m}$.

The internal tide carries energy away at a rate of $E_{IL} c_{gIL} + E_{IR} c_{gIR} = \frac{1}{2} g' \zeta_l^2 c_{IL} + \frac{1}{2} g' \zeta_r^2 c_{IR} = 0.5 * 9.81 * 1/1025 * 13.9^2 * 1.27 + 0.5 * 9.81 * 1/1025 * 14.0^2 * 0.95 = 1.2\mathrm{kW/m}$.

This gives an idea that the internal tide only removes ¡1% of the surface tide energy at this step, somewhat justifying our assumption that the surface tide is not strongly affected but the internal tide.